WB-09

LOW PRESSURE EXPERIMENTS
AND
MODELING
FOR
HIGH SCHOOL SCIENCE CURRICULA

Science and Technology
of
Materials, Interfaces and
Processing

A STEM WORKSHOP CONDUCTED BY
THE EDUCATION COMMITTEE
AVS SCIENCE & TECHNOLOGY SOCIETY

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A STEM Workshop for High School Science Educators

Conducted by

The Education Committee
AVS Science and Technology Society

PREFACE

The AVS Science and Technology Society is a nonprofit organization which promotes communication, dissemination of knowledge, recommended practices, research, and education in technologies that require a vacuum and other controlled environments. It publishes research in these topics and conducts various local and national educational programs for engineers, scientists and technicians The Society recognizes the national impact of a technical workforce that keeps pace with the new technologies developed by its members. The AVS supports numerous outreach activities aimed at teacher enhancement and student science awareness and education. Through these various outreach programs, the Society is able to share the collective and individual expertise of its members with schools, teachers and students.

The workshop associated with this workbook emphasizes the fundamentals of science, technology, engineering, and mathematics (STEM) and how they integrate into science curricula. Emphasis is made of the critical role high school teachers have in the influence of student's attitudes, interest, and fundamental perception of science and technology. Career opportunities in science, technology and engineering are also introduced.

The workshop is divided into two parts:

1. lectures on the underlying physics concepts and the mathematical calculations related to vacuum technology; and
2. hands-on experience in which small groups of science educators perform experiments and develop models under instructor guidance

The workshop experiments are designed to relate vacuum technology with everyday phenomena, familiar to students. This workbook supplements and compliments the workshop activities. The book contains instructional and application-oriented materials to focus participant’s interest on vacuum related engineering science concepts that can be developed for classroom use.

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I. INTRODUCTION

There is a perception that science and engineering are distinctly different disciplines. Scientists wonder "why" and engineers "how". In fact, the thinking of scientists and engineers is more alike than different in the current technological world. In the area of vacuum science and technology the enabling characteristics have always made the distinction between the two difficult. For example, were Edison’s efforts to improve vacuum pump technology driven by the desire to explore low pressure environments or solve the inventor’s problems associated with filament failure? Is the international space station about science in a vacuum environment or technology that will be of value a couple of miles below the station?

The focus of this workshop is the different aspects of the natural and physical sciences as they relate to observations of different pressure environments generated with a vacuum system. Attention is directed to engineering perspectives of these phenomena and the impact they have had, and continue to have, on our lives. The “hands on” format interspersed with short lectures and group exercises provide a variety of leaning experiences.

The overall goal is to provide a different perspective of the interaction between science and engineering. This workshop relies on the premise that predictable events occurring when the pressure of an environment is manipulated are interesting to students. It begins with a general introduction to technologies that rely on a vacuum (low pressure) environment. This is followed by a review of popular demonstrations and exploration of new applications for vacuum in the high school science curriculum. The workshop concludes with an overview of vacuum equipment, including operation, maintenance, and safety precautions.

In summary, the workshop deals with the science and engineering aspects of systems at pressures other than one atmosphere. Pressure and temperature provide a portal to many phenomena in nature. Pressure indicates the force applied to a substance per unit area of that substance and suggests the influence that force has on molecules in the surroundings. Temperature reflects the internal energy of a substance and the transfer of that energy to other substances. Changes in either or both of these thermodynamic properties reflect a new state for the substance and its environment that can spark an observer's curiosity.

II.   LOW PRESSURE TECHNOLOGIES

Low pressure technologies are the backbone of today's "high tech" society. Low pressure is a relative term that implies a pressure environment that is below normal sea level atmospheric conditions. The earth is surrounded by a mixture of gases referred to as the "atmosphere". This atmosphere, as illustrated in Figure 1, extends from the earth's surface to approximately 1000 km, with most of the atmosphere below 10 km. Up to 10km altitude, its composition is primarily nitrogen (78%) and oxygen (21%). The remaining one percent contains Ar, CO_2, He, Kr, Xe, CH_4, H_2 and H_2O. All gas molecules in the atmosphere are in constant motion, undergoing millions of collisions with other gas molecules or surrounding surfaces every second. The result of these collisions is the exertion of an average force per unit area in all directions. At sea level and 25 ^{0}C, this force per unit area is defined as 1 atmosphere of pressure and is approximately 100,000 Newtons per square meter (1 Pascal) or 14.7 pounds per square inch. The internationally accepted non-geographic specific definition of one Atmosphere (1 Atm) is the pressure exerted by a 760 mm column of mercury having a specific gravity of 13.595 g/cm^3 at 0^{0}C.

One of the first recorded attempts to measure atmospheric pressure was Otto Von Guericke's famous Magdeburg hemisphere experiment reported in 1672. The attempt certainly employed the "high tech" equipment of the period. In his experiment, von Guericke tried semi -unsuccessfully to separate two evacuated hemispheres with two teams of horses. The vacuum pumps used to remove the gas from between the two contacting hemispheres were created from bellows similar to those that might be used today to fan a stubborn campfire. As with many of today's high tech experiments, Von Guericke's inability to create the proper materials lead to the final victory for the horses. However, the horses were not really able to pull the evacuated hemispheres apart until the "best at the time" seals between the hemispheres allowed too much air to leak back into the evacuated sphere.

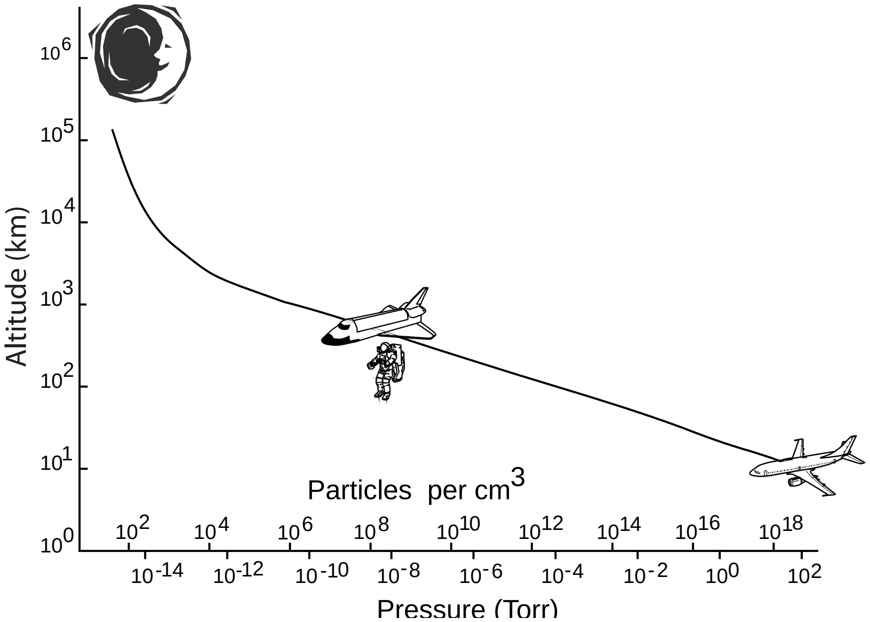


Figure 1. Altitude above the earth's surface vs. pressure and particle concentration.

Even though the experiment was only a partial scientific success its showmanship captured the audience and provided a startling demonstration of the force associated with an evacuated chamber. It will have the same effect in your classroom today. This experiment mixed with a bit of showmanship and bravado will have the same startling and entertaining effect in your classroom today as it did to its audience in 1672.

A complete list of low pressure technology applications is actually much longer than the partial list in Table 1. The entries provided emphasize the diverse aspects of the products that result from low pressure science and technology. In fact, it is tempting to suggest that there is no connecting scientific principle to all of the technologies listed. This is not the case, however. Vacuum science and technology are a common thread running through these low pressure technologies. Figures 2 and 3 show examples of "high tech" products that rely on low pressure environments for their manufacture.

The connection among these technologies is the molecules or lack of molecules involved in the process and their energy content as it relates to the temperature and pressure in their associated environment. In the gas state, the energy associated with a set of molecules reflects their random motion characteristics which, in turn, is key to describing the pressure on the environment associated with those molecules. The kinetic theory of gases provides an aid to visualizing the influence molecular type and motion has on the pressure of a system. One of the main assumptions of the kinetic theory of gases is that gas molecules are in constant motion. The velocity at which various gas molecules travel reflects their kinetic energy and is directly dependent on their temperature and inversely dependent to their mass. Heavier molecules move slower than light ones and all molecules move faster if the temperature is increased. However, from a thermodynamic perspective, a mixture of different types of gas molecules will exert a higher pressure if the number of molecules or the temperature of any or all of the molecules is increased. The number of molecules involved and their molecular velocity are collectively represented by the pressure they exert on their surroundings. Thus, low pressure technologies are commonly successful because, for the process in question, the molecular density has been reduced and the molecular velocities curtailed to a process -specific range of values.

T=he complete list of successful low pressure technologies is long and impressive. Figures 4-7 show four low pressure systems that are used for manufacturing processes and materials research. Today's society routinely depends on products manufactured in a controlled vacuum environment. The semiconductor industry is just one typical example. A microprocessor chip is created by a series of energy controlled chemical reaction steps that involve minimal collisions with contaminated molecules. Since the distance that molecules can travel before a collision reflects the pressure that is exerted by these molecules, this mean free path must be long if the reaction conditions for successful microprocessor chip manufacture are to be accomplished. At atmospheric pressure the mean free path is extremely short, about 6×10^{-5} mm. Reducing the pressure increases the mean free path distance dramatically and removes one of the major barriers to the semiconductor manufacturing industry.

Naturally, the mean free path requirements vary for specific low pressure technologies. The range of pressures for various technologies is provided in Figure 8. The examples range from electron-beam welding at 1000 Torr, pressure conditions that are 240 Torr above normal sea level pressure to materials research at 5 pico Torr, the pressure conditions that exist in outerspace. Figure 9 shows a thin film deposition experiment carried out in low earth orbit.

Table 1. Low pressure technologies

- automotive - carburetion and controls
- biological - pharmaceutical distillation
- braking - trains
- cathode ray tubes
- environmental controls and cleanup
- food procession
- information transfer
- lighting
- materials characterization
- materials handling
- materials processing
- medicine
- refrigeration
- space exploration and sciences
- semiconductors manufacture
- thin film growth
- vacuum coatings and brazing

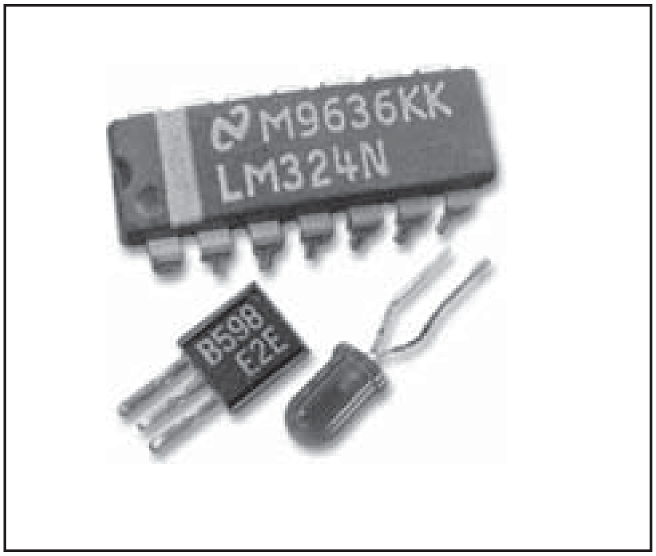


Figure 2. Semiconductor devices

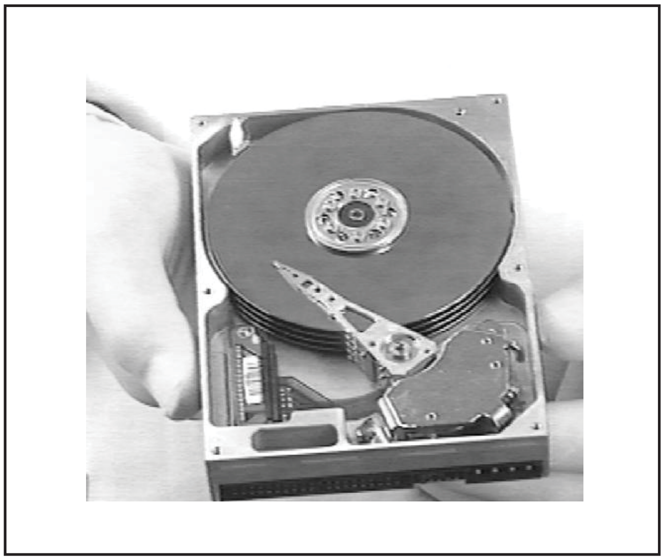


Figure 3. Computer hard drive assembly.

Vacuum Research and Processing Equipment

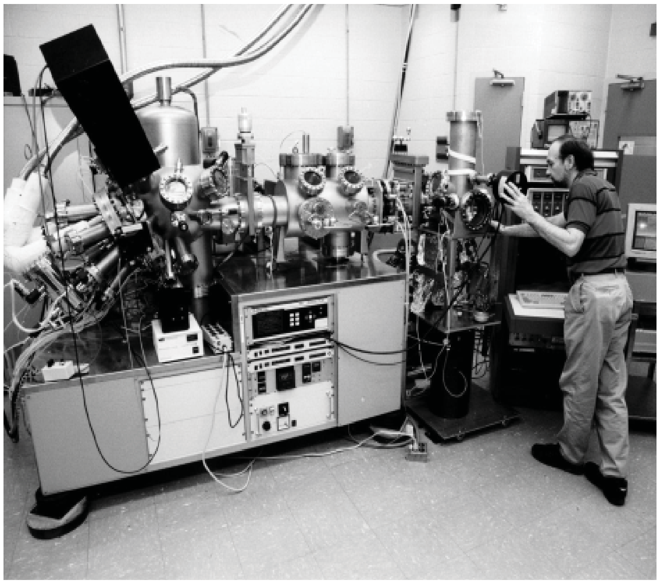


Figure 4. Ultrahigh vacuum molecular beam epitaxy thin film deposition system.

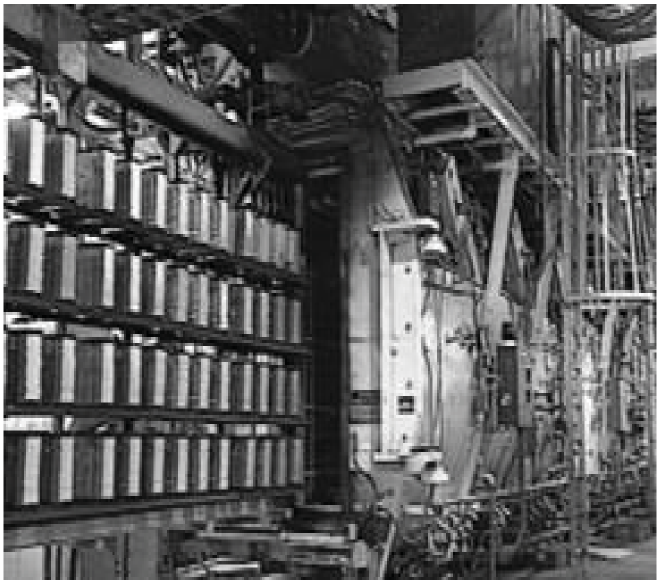


Figure 5. Large vacuum brazing oven for automotive air conditioning parts.

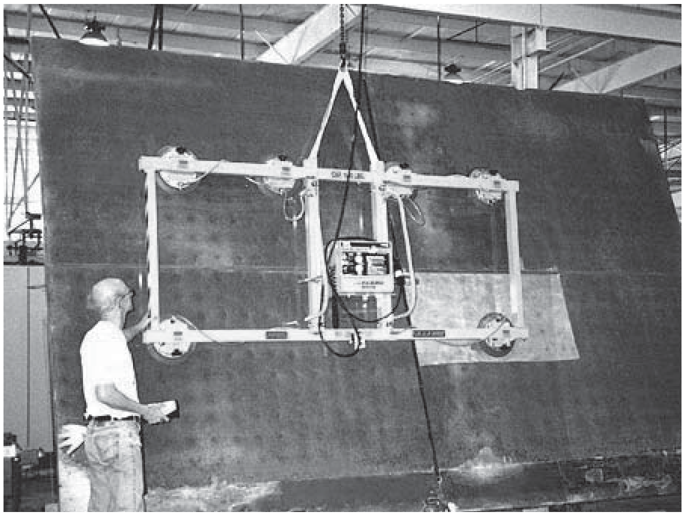


Figure 6. Vacuum lifting tool.

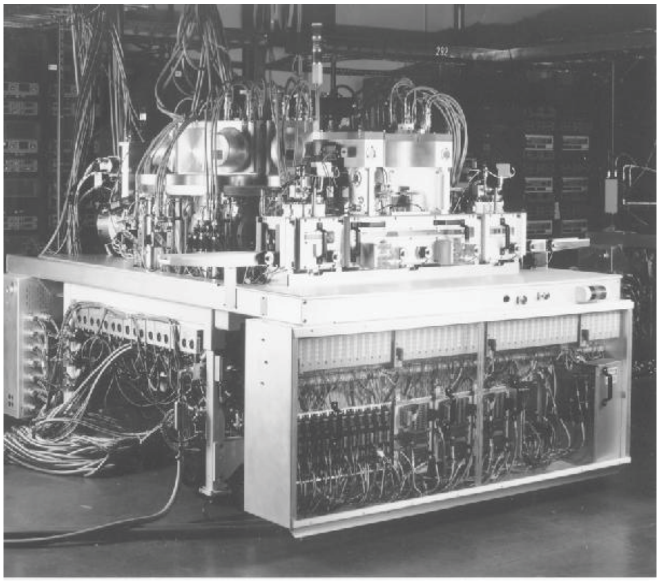


Figure 7. Semiconductor vacuum processing unit.

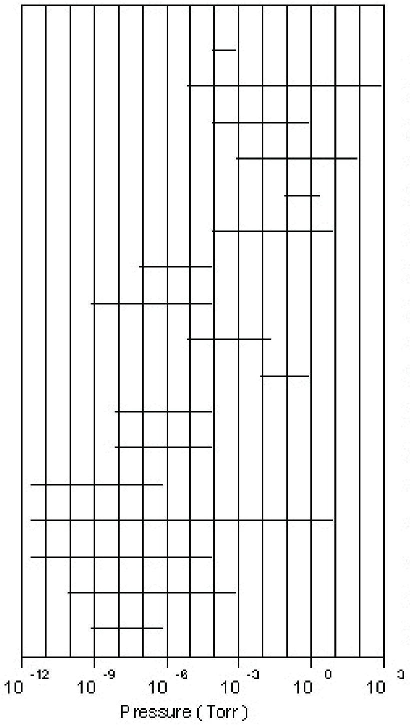


Figure 8. Pressure ranges for low pressure technologies

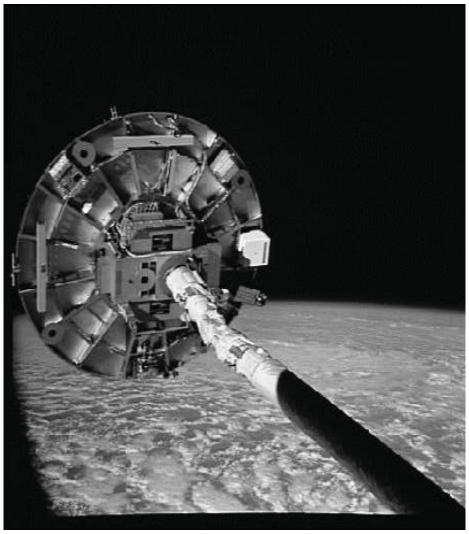


Figure 9. Low earth orbit molecular beam epitaxy thin film deposition unit.

III. CONTROLLED ENVIRONMENT EXPERIMENTS

Any classroom demonstration has to have a specific set of characteristics to assure its success with its intended audience and still meet the educational objectives of the science educator. First, the demonstration must be easy to assimilate in a group situation. This not only implies that there are no visual or audio impediments embedded within the demonstration but that the information registered by the observer's senses actually makes sense to the observer. Next, the results of the demonstration must provide the observer with insight into but not necessarily answers about the phenomena associated with the demonstration. Finally, the demonstration must be safe, inexpensive and repeatable if the sanity and nervous system of the educator is to remain sound.

Some science demonstrations are unique in that the same demonstration possesses the three characteristics presented above even when the observers are grouped by age. Many vacuum based classroom demonstrations reflect this quality. A list of vacuum environment classroom demonstrations is provided on the following page. The marshmallow demonstration is a classic that is suitable for all grades from the third grade to the tenth grade. The difference, of course, is the intent of the demonstration and the expectations of the observer. For the fifth grader, the initial expansion of the marshmallow provides the entertainment and impact of the demonstration's intent, and the importance and power of normal but invisible atmospheric pressure. While the tenth grader, after an initial giggle over the marshmallow's "Ghost Busters" expansion, will be more focused on the reasons for the marshmallow's contorted final fate as the pressure continues to drop.

All of the demonstrations suggested listed in Table 2 meet the requirements of a good classroom demonstration. Many of them will work with a variety of age groups. None of them will be successful unless some attention is paid to the vacuum system that supports the demonstration. An amazing thing about vacuum systems is the fact that they are shunned by most science educators but are, in fact, a safe and easy-to-use educational tool. The reluctance to use a vacuum system in the classroom stems most often from our natural tendency to avoid using equipment that is unfamiliar and uncomfortable to us. However, the vacuum system's case, there really is not much to become familiar with. These systems are quite durable and dependable with very little bit of TLC. The simple system illustrated in Figure 10 is actually a perfect match to the technology needed for most vacuum science demonstrations and experiments for all grades, K through 12. The diagram shows a vacuum chamber that is connected to a pump. The pump is run by an electric motor, the pressure environment inside the vacuum chamber is monitored by a gauge, and a valve is used to return the system to atmospheric pressure.

In this workshop, different pumps, shown in Figure 11, are available to perform the various demonstrations and experiments. Although the connections between these pumps and vacuum jars may be different, the basic system configurations with these pumps are all similar to that shown in Figure 10.

Table 2. Classroom demonstrations that use vacuum

1. adiabatic expansion √
2. Archimedes Principal
3. atmospheric pressure
    - a. marshmallows √
    - b. vacuum plates √
    - c. pump speed √
4. electron emission
5. gas laws - Boyle's law √
6. gravity
    - a. falling feather √
    - b. acceleration of gravity
    - c. drag force
7. heat transfer
8. index of refraction of air
9. ionization of gases
10. latent heat √
11. sublimation of ice
12. pressure effects on a pendulum
13. refractive index of air and other gases
14. thermionic effect - measuring pressure
15. thin film growth
16. transmission of sound √
17. vapor pressure of water
18. weight of air
19. speed of sound as a function of pressure

√ indication of demonstrations and experiments covered in work shop

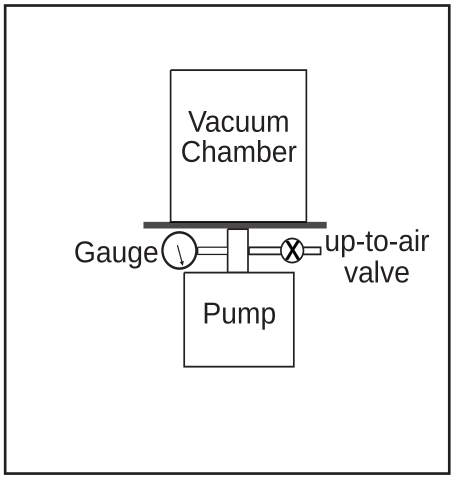
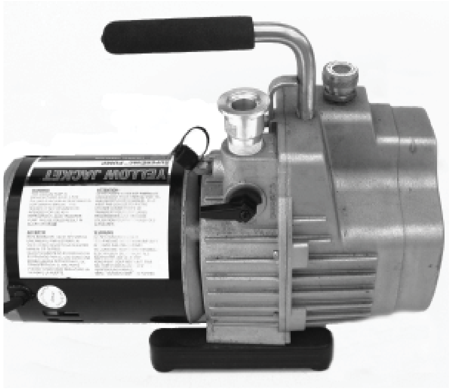
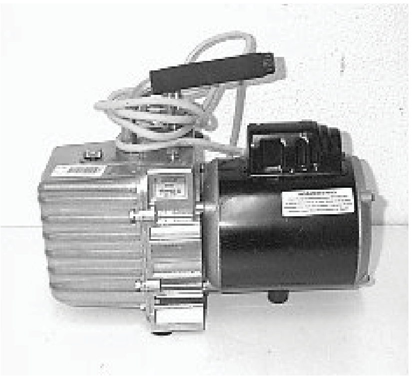


Figure 10. An illustration of a simplevacuum systems for classroom activities, demonstrations, and experiments.



Yellow Jacket SuperEvac
Ritchie Engineering Co., Inc.
www.yellowjacket.com

Robinair 15400 Cool Tech
Robinair / SPX Corporation
www.robinair.com



J/B DV-142N
J/B Industires
http://www.jbind.com/

Figure 11. Vacuum pumps used in the workshop.

Demonstration 1: Expanding Balloon

Inside a deflated balloon, the pressure is about the same as the surrounding atmosphere. To inflate the balloon, the pressure inside is increased by forcing gas into it. The expansion of the balloon is opposed, partly by the material of which it is made and partly by the pressure of the atmosphere acting on the outside of the balloon. If the pressure of the atmosphere on the outside of the balloon is removed, the only requirement would be to stretch the balloon material.

To demonstrate, tie off the neck of the balloon, leaving a little air at atmospheric pressure trapped inside . Place the balloon in the vacuum jar, as illustrated by the smallest circle in Figure 12, and evacuate the vacuum jar. As soon as the air removal begins, the balloon begins to expand, since the pressure inside now exceeds that of the surrounding air. When the pump has removed all the air it can, the size of the balloon will depend on the volume of the air in it and the resilience of the balloon material.

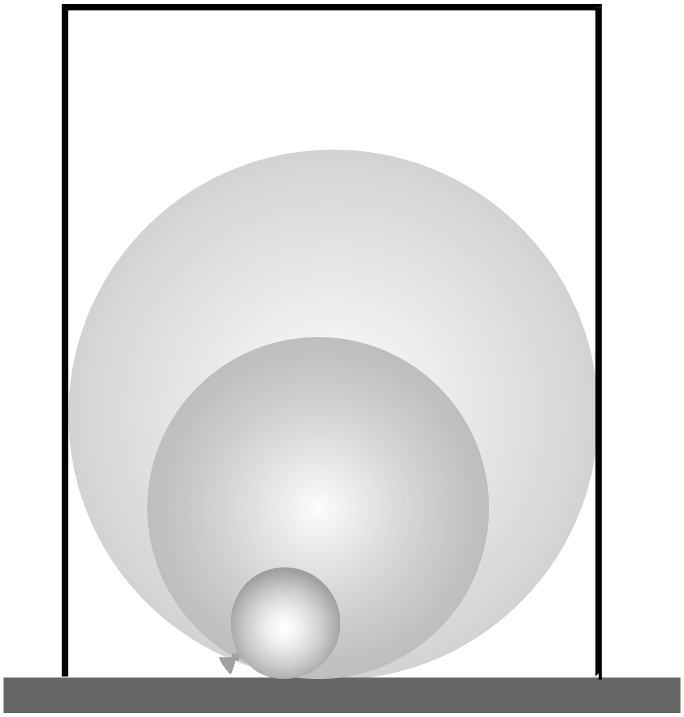


Figure 12. Expanding a balloon in a vacuum chamber.

Demonstration 2: Falling Feather

The velocity of an object moving through air is impeded by the air resistance. At low velocities the resistance is normally insignificant; at high velocities the retarding force can be important as in aerodynamics. All moving bodies feel a certain resistance as evidenced by watching a feather or a piece of paper fall to the ground. This effect can be demonstrated by observing an object falling a fixed distance in both atmosphere and vacuum. To demonstrate, a 'light' feather and a 'heavy' piece of metal are dropped simultaneously in the vacuum jar which is alternately at atmosphere and low pressure. The feather, being affected more by the air pressure will not fall as quickly as the piece of metal in air. However, in the evacuated vacuum jar, where the retarding force due to the presence of air is removed, the rate of fall will be the same for both the feather and the piece of metal.

Practical Points

(i) Use a piece of metal that is magnetic. Suspend it on the inner topside of the vacuum jar using a magnet as shown in Figure 13 with a feather partially sandwiched between the metal and the inner face of the vacuum jar.
(ii) The feather should be extremely light to enhance the effect since the distance that the objects fall is not very great. Down feathers work best.
(iii) Do not sandwich the feather completely between the metal and vacuum jar face - they might stick together nulling the effect. Remove the magnet quickly when demonstrating.

Discussion:

Discuss the effect of the mass of the falling body on its drop time. Distinguish between total force and force per unit area (why does a parachute work?). Check out the advanced experiments section at http://www.avs.org/education.workshop for an advanced experiment on the acceleration of gravity.

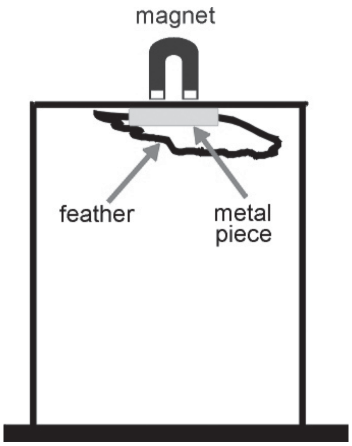


Figure 13. Setup for observing a falling feather in a vacuum chamber.

Demonstration 3: Transmission of Sound

Sound transmission requires a medium, such as a gas, liquid or solid. In air or any other gas, sound is transmitted by collisions between the molecules making up the gas. The molecules close to the source of the sound move in unison with its vibration. The movement of the molecules has the same amplitude and frequency as the source. This motion is transmitted by successive collisions between these molecules and other gas molecules, resulting in a wave-like transfer of vibrations outward from the source to a receiver, with a gradual decrease in the amplitude of the vibrations. This means that if no air or other gas is present, the sound will not be transmitted.

The effect of air pressure on sound transmission can be demonstrated with any number of sound producing devices, such as a mechanical alarm clock with external clapper, a radio, or a bursting balloon . Regardless of the device used, however, it is important to provide as much vibration dampening as possible between the sound source and the vacuum chamber since any solid - solid contact can also act as a transmission medium. A setup for demonstrating the absence of sound in vacuum is shown in Figure 14.

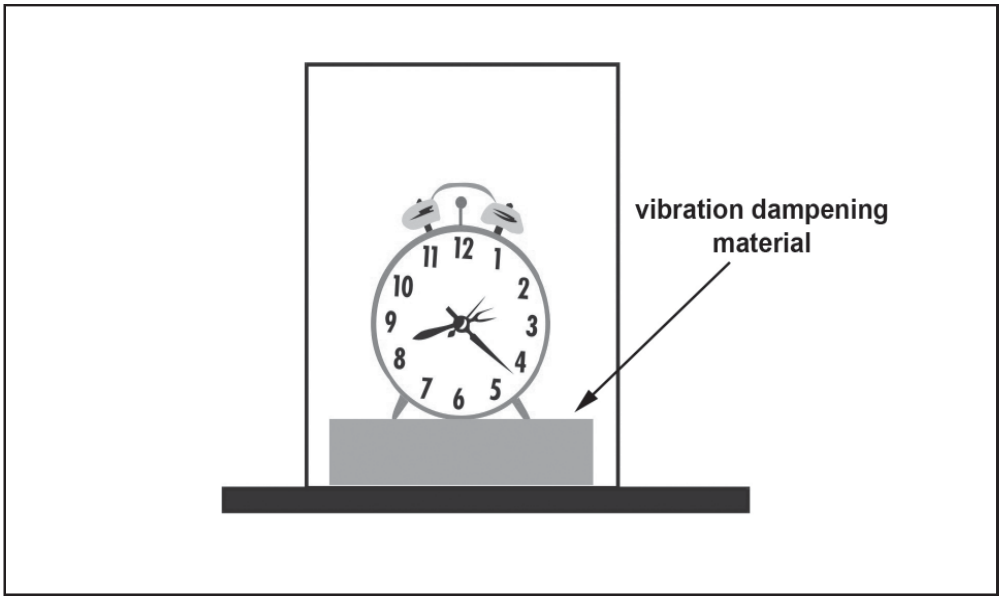
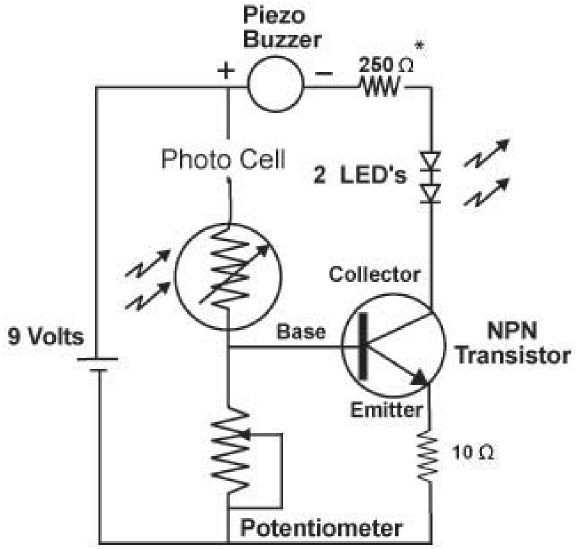


Figure 14. Setup for demonstrating the absence of sound in a vacuum.

A very effective device for demonstrating the difference between sound and electromagnetic radiation transmission is a battery powered photo sensitive device with light emitting diodes and a sound emitting piezo buzzer that can be controlled outside the vacuum chamber. With such a device the point can be made that while sound transmission is dependent on a gas, liquid or solid medium, electromagnetic radiation such as heat, light or radio waves require no medium for transmission. A schematic diagram for such a device is shown in Figure 15. With such a device, a demonstration could illustrate that sound transmission uses a gas or solid medium, electromagnetic radiation (such as heat, light and radio waves) requires no medium for transmission.

The circuit is simple to build from parts available at Radio Shack and is easy to explain to students who have some knowledge of the voltage and current concept. It uses a 9 volt battery to supply operating energy to the photo cell, the light emitting diodes (LED's), and the piezo buzzer. The potentiometer that is serially connected between the photo cell and the battery terminal adjusts the voltage and current conditions at the base of the NPN transistor. The base input, in turn, determines if there is an adequate connection between the emitter and collector terminals of the transistor. If that connection is met, then the five LED's will light up since they will now be connected across the 9 volt battery. Naturally, the piezo buzzer will also vibrate since it is connected in parallel with the LED's. The vibrating buzzer will be easily heard at normal atmospheric pressure. The connection between the emitter and the collector of the transistor will support the current flow necessary to turn on the piezo buzzer when light is shown onto the photo cell.



PARTS

Piezo Buzzer	Radio Shack 237-060
Photo Cell	Radio Shack 276-1657
LED	Radio Shack 276-316
Potentiometer	Radio Shack 271-280
Battery, 9V and holder	
Transistor (NPN)	Radio Shack 276-2016
Resistor	Radio shack 10 ohm 271-1301
Resistor	Radio Shack 250 ohm
Circuit board	Radio shack 276-174

* Value of resistor will depend on number and resistance of LED's

Note: For best results, shield the photo cell from indirect light and adjust the potentiometer in room light for no sound.

Figure 15. Schematic diagram for a sound emitting device controlled by a visible light source.

IV. WORKING WITH MODELS

The best quality of STEM as an education vehicle is the fact that it generates a great feeling when a concept is completely understood in terms of the science, technology, engineering, and math involved in the concept. This kind of success is actually a three step process in which all three parts are equal in value: theory, model, and observation. (See Figure 16.) When a physical phenomena is completely understood it can be explained by a satisfactory theory, successfully described by a mathematical model and characterized by repeatable observations.

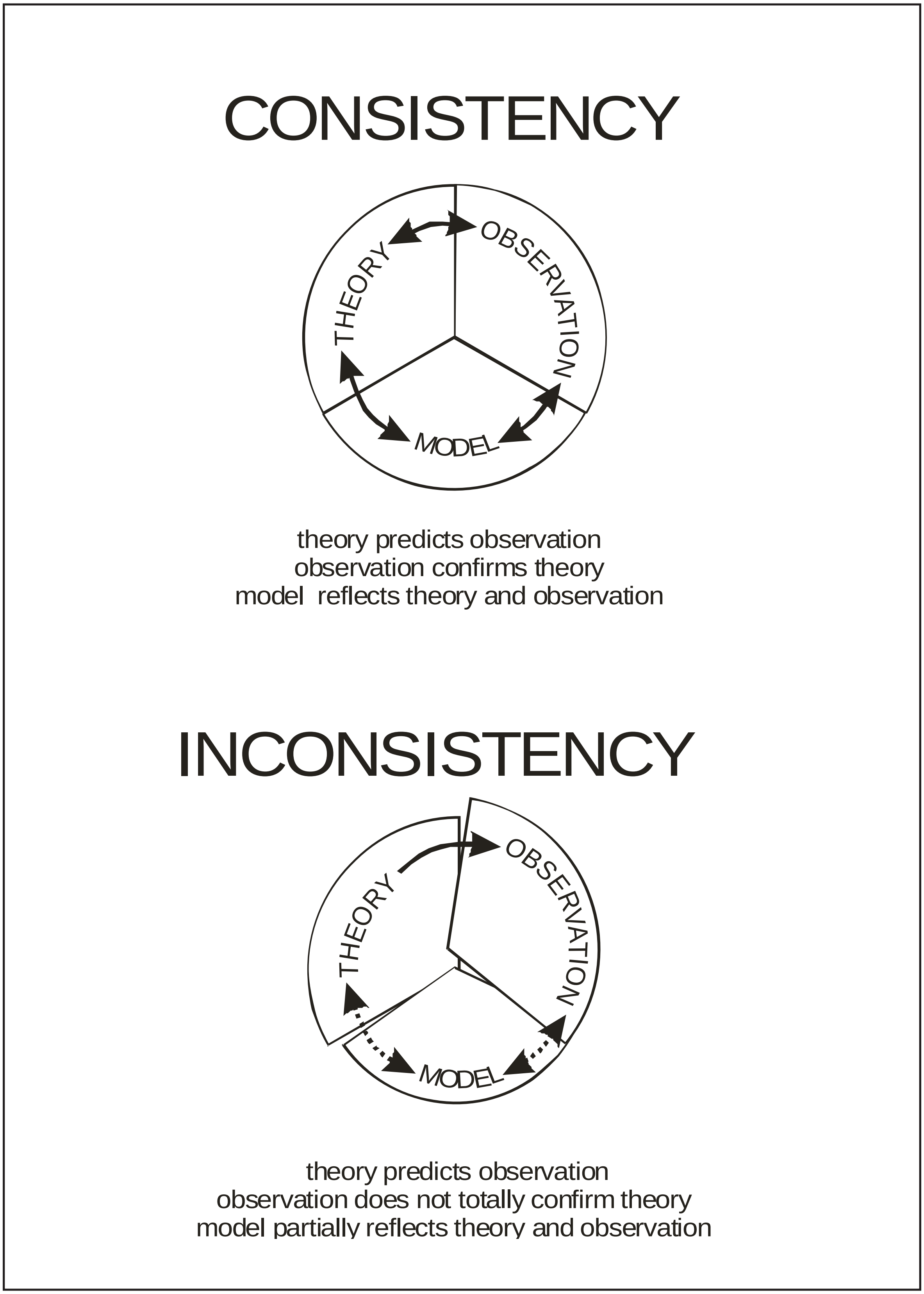
Although an attempt to completely understand any concept in the natural or physical sciences requires success in all three of the puzzle part steps, usually it does not matter which of the three steps is attacked first. In fact, there are many important historical examples where each of the steps, theory, observation, or model were first in the development of the understanding of a scientific principle. There is, however, a recognized procedure after the process has begun. This process involves cycling and recycling through each of the three steps until there is a recognized link between theory, the observation and model for the phenomena in question.

For example, if the quest for knowledge begins with the observation step it is common for many repeated observations to be made to confirm that initial observations are valid and precise. The next phase is often to find a mathematical model that will describe the observations and perhaps facilitate predictions about additional experiments to confirm the observations. Finally, the third step in this case, is to develop a theory that complements the model and the observations. Once this process cycle has been completed and the conclusions are acceptable (based on the current state of knowledge about the subject), the theory, with or without some alterations, may be used to enter a new cycle for a different but related scientific phenomena that is subsequently characterized by experimental observations and ultimately described by mathematical model or models.

IV(A). The Role of the Model

This workbook uses the term “model” to refer to the set of mathematical statements, equations, or functions that are intended to quantify a phenomena of interest. By contrast, theory is a set of verbal images that facilitate the description of the phenomena to others. For example, the kinetic theory of gases is the description of molecular activity as it relates to energy and motion. There are many excellent verbal descriptions and predictions of molecular behavior as proposed by the kinetic theory, and there is a specific set of mathematical relationships and equations that quantify these descriptive statements. The theory of ideal gas behavior is one subset of the kinetic theory of gases and comes with its own observations and models.

Ideal gas behavior is centered on three assumptions. First, that molecules in the gas state are spherical in nature. Second, molecular collisions are completely elastic. Third, there are no chemical or electron interactions between different types of molecules when they share the same volumetric space. In this case, the model is a classic mathematical equation, $PV = nRT$, and is used to quantify the behavior of any ideal gas. This equation, i.e., the model, translates the verbal and mental images of the theory into a set of tangible parameters that can be observed and measured. Thus, verbal images of the theory and model can be quantified and ultimately observed to confirm this model’s validity.



CONSISTENCY

theory predicts observation
observation confirms theory
model reflects theory and observation

INCONSISTENCY

theory predicts observation
observation does not totally confirm theory
model partially reflects theory and observation

Figure 16. The three step process needed to completely understand a physical phenomena.

Granted, the distinction between theory and model provided above has an arbitrary tint to it. Most of the time it is easy to internally separate the theory from the model and proceed with the exploration of the phenomena of interest. There are few times when the person(s) doing the investigation really has to stop and describe the actual three step process (theory, model and observation) that is occurring. The only significant result is related to the success that the person has connecting the three steps. However, for students who are new to this way of thinking about scientific ideas, it is certainly helpful to separate the steps of the process so that they can recognize each phase of the puzzle-solving procedure. Since working with any of the steps of this process is equally difficult to students, in the long run it does not make any difference which is tackled first. However, it is very important that students are exposed to all three aspects of this scientific method as well as their ultimate synergy if they are to successfully assimilate the complete process and make it their own.

IV (B). Skeleton of Some Common Models

The incredible legacy of the 19th century scientific and engineering community is based on the fact that most of the engineering and science principles studied lead to fantastic inventions and corresponding technologies that were best modeled with simple mathematical expressions. This is a mixed blessing for scientists and engineers of the 21st century because the habitual practice of using a linear, $y = mx + b$, model to describe a phenomena is ubiquitous. On one hand, such modeling is an excellent way to quantify a theory based on the expectations of a single behavior or variable. On the other hand, phenomena under study in this century depend on the changing behavior of many variables simultaneously. In any event, it is clear that students must understand the phenomena studied in the past if they are going to keep pace with the discoveries of the future. Thus, exposure to linear models is important to their developing thought processes.

The late 18th, the 19th, and to some extent 20th century habit of making a theory fit this linear relationship is easy to understand. First, in many cases the linear model was the best choice. Second, without the aid of calculators and computers the linear model was the practical way to proceed. Third, most of the measurement devices available at the time of the verifying experiments are crude compared to those available today. Indeed, scientists and engineers become so captivated with the success of linear models, they often develop mathematical ways to transform nonlinear equations for presentations as linear representations. Taking the natural log of an exponential expression is the classic example of this linearization process. The application of a Laplace transform is yet another. Although discussion of the latter is beyond the scope of this workbook, the former is not. In fact, the exponential equation is the model component of a great number of scientific theories.

$$\boxed{e^{-[(\frac{a}{b})\chi]} = 1/e^{[(\frac{a}{b})\chi]}}$$

$$1 \longrightarrow 0$$

$\chi = 0$ $\qquad\qquad$ $\chi = \infty$

key strokes along χ when the value within [ ] is an integer

i.e. $[(\frac{a}{b})\chi]$ = 0, 1, 2, 3, ...

$$e^{-[0]} = (1/e)^0 = (1/2.72)^0 = 1.00$$
$$e^{-[1]} = (1/e)^1 = (1/2.72)^1 = 0.37$$
$$e^{-[1]} = (1/e)^2 = (1/2.72)^2 = 0.14$$
$$e^{-[3]} = (1/e)^3 = (1/2.72)^3 = 0.05$$

[value within brackets must be unitless]

Figure 17. Skeleton form of the exponential decay model

The exponential expression is based on the fact that the answer (the variable of interest) is related to the question (the variable that is easy to observe) by a power relationship with a third number, the base. For example, the dollar, certainly a variable of high interest, is related to a number of pennies, a variable that is observed in any sales tax transaction, by raising the number 10 to the second power. That is to say, you need 100 pennies to equal the value of one dollar. Although exponential expressions using 10 as the base are common in science and engineering, expressions using the number 2.72... is an even more popular base number. This number, usually represented by the letter "e", creates exponential expressions that are extremely common in science and engineering. These expressions are easy to linearize using a natural logarithm operation. As such, it is certainly important to understand some of its fundamental characteristics. And, these fundamentals are simple.

Fortunately, there are really only a few key images needed to transmit the essence of an exponential expression to students. The first is illustrated on the top of the Skeleton Model of Exponential Decay illustration in Figure 17. This graphic emphasizes the fact that a negative exponential expression, no matter how complicated the terms in the exponential part are, is ultimately a number that has a value between zero and one. This information, although extremely important to remember, is often forgotten or never realized by students first working with exponential functions. To further demonstrate this point, Figures 18 and 19 illustrate specific ranges of the exponent parameters for the exponential growth and decay models. The shapes of these curves should be recognized by students.

The second useful generalization about the exponential decay model is shown in the bottom section of Figure 17. Here the emphasis is not on the range of values the exponential decay model may have, i.e., from one to zero as x goes from zero to 1, but the fact that there are popular values of x that are inserted into the exponential decay model. These particularly useful x values are the values that will make the entire exponent for the exponential decay model become integers. For example, an interesting x value for any exponential decay model is the x value which makes the complete exponent expression equal to one, giving the entire exponential decay model a value equal to 0.37. The illustration also indicates what happens when the x value inserted into the exponential expression makes the exponent equal to 2 or 3. In these two latter cases, the entire exponential decay model takes on a value of 0.14 or 0.05 respectively. These three numbers, 0.37, 0.14 and 0.05 are found in a multitude of places in every branch of science and engineering, explaining and supporting many theories about a variety of naturally occurring and forced phenomena.

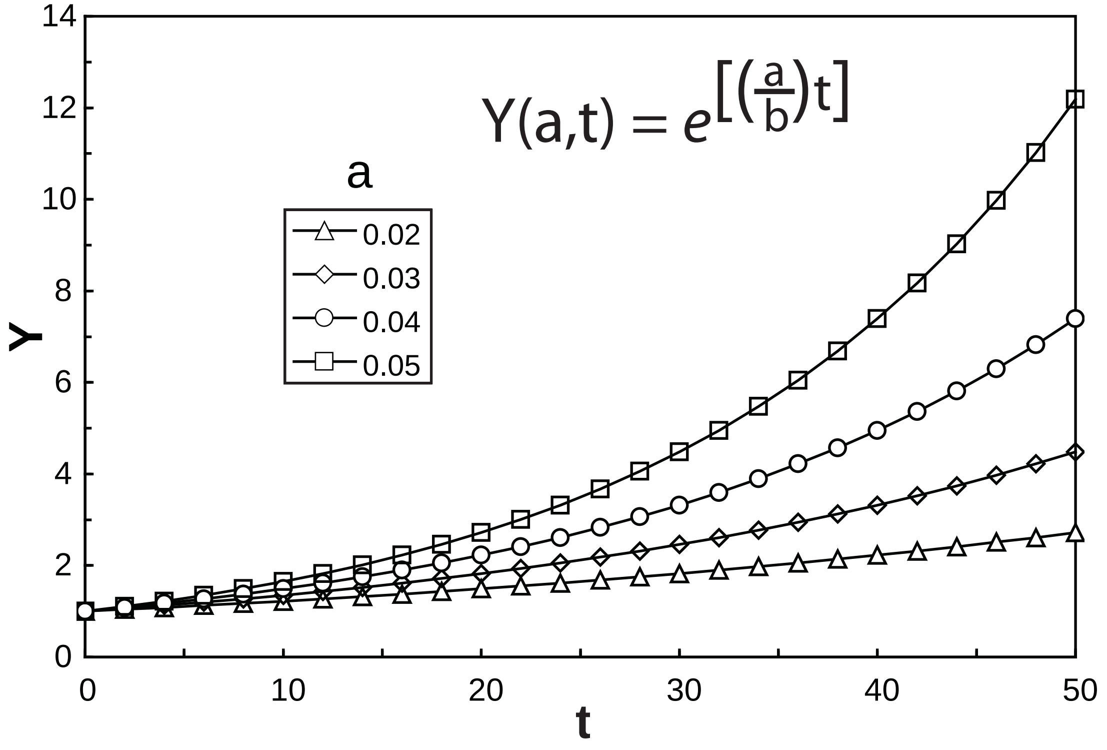


Figure 18. Exponential growth (family of curves with b=1).

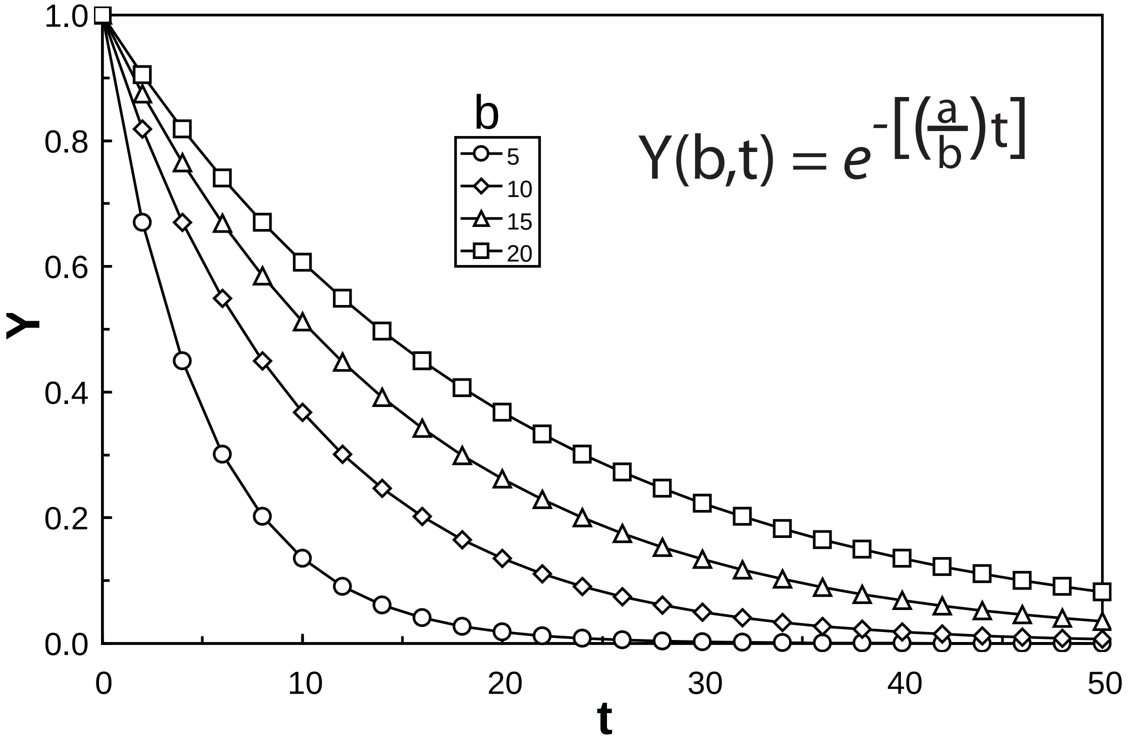


Figure 19. Exponential decay (family of curves with a=1).

IV(C). Applications of the Exponential Model

The exponential model is ubiquitous in the physical sciences and engineering. A good way to bring this fact to the attention of students is to engage them with a scavenger hunt. Send them off to the library and/or the web to find such examples. In addition, ask them to determine if the definitions and meaning of the terms are in the exponent of the exponential expressions they find. For example, if a student explores electrical engineering topics the exponential model will be quickly found. It is used to describe what happens to voltage as a function of time in a circuit that has a resistor and a capacitor in series. In this case, the “b” in the denominator of exponent term represents the resistance and capacitance of these two serial circuit components. The time value, t, which makes this exponent term, t/RC, equal to one is identified as the time constant for that resistor capacitor circuit. This specific value of t corresponds to the time necessary for the voltage across the capacitor to decay from its maximum possible value to 0.37 of that value.

Table 3. Scavenger hunt

Materials Science	Diffusion in Solids
Mechanical Engineering	Pumping Speeds for Gases
Environmental Engineering	First Order Reaction Rates
Electrical Engineering	RC Charging
Industrial Engineering	Statistical Distribution
Nuclear Engineering	Radioactive Decay
Others	? ....

V. DEVELOPING MODELS

Since models are valuable for working with physical systems, it is useful to introduce students to the task of developing models. There are two general routes to model building. The first is to develop the model from the theory considerations. The second approach is to develop the mathematical model from data that quantifies the observations of the phenomena under study.

V(A). Building a Model from Theory

A vacuum system consisting of a pump with chamber provides an excellent opportunity to have a student test a model that was developed from a theory. In this case, ideal gas behavior as developed from the kinetic theory of gases will be used. In addition, students can test the model developed from this theory with a very simple experiment that addresses the question:

> "How long does it take to reduce the gas pressure inside the vacuum chamber
> from atmospheric pressure to a specific value below one atmosphere?"

To begin, consider air to be an ideal gas and visualize its presence in the vacuum chamber exerting atmospheric pressure. Once the vacuum pump is turned on, this air begins to leave the vacuum chamber because it is being pulled into the pump and then pushed into the surrounding air outside the vacuum chamber. Naturally, the number of gas molecules inside the vacuum chamber decreases as a function of time. The longer the pump keeps pumping the more gas is removed from the vacuum chamber. Thus, the question of interest really reflects how fast the pump can remove molecules from the vacuum chamber. The pump must be able to remove molecules from the chamber faster than molecules can return to the chamber. In other words, the pressure in the chamber will stop dropping once the number of molecules the pump is removing per unit of time is equal to the number of molecules per unit of time entering the chamber via leaks in the entire vacuum system.

The ability of a pump to remove molecules from the vacuum chamber is expressed by means of two parameters. One parameter is called throughput, Q, of the pump while the other is the pumping speed, s, of the pump. The throughput of a pump is a value that represents the number of molecules per unit time the pump can remove from the chamber. The pumping speed is the volumetric flow rate (volume per unit of time) the pump can maintain. To secure an answer to the question above, we can use a model that includes the pumping speed and expresses the current pressure inside the chamber as a function of time the pump has been running.

Student Homework Problem V(A)-1. Pump Throughput

Develop an equation using the Ideal Gas Law from the kinetic theory that will relate a vacuum pump's throughput, Q, to the variables in the Ideal Gas Law , which is expressed as follows

$$N = PV/RT$$

where N is the number of molecules, P, V, and T are the system pressure, volume, and temperature, respectively and R is the gas constant.

Answer:

If the only way molecules enter or leave the vacuum chamber is through the vacuum pump, and if no molecules enter the vacuum system while the pump is on, then the number of molecules, N, that exit the chamber and enter the vacuum pump per unit of time becomes N/t. In addition, the number of molecules in the vacuum pump at any instant of time can be calculated using the Ideal Gas Law with the system being the vacuum pump.

Therefore N/t, which is the definition of Q, becomes:

$$Q = N/t = (PV_{pump}/RT)/t = (P/RT)(V_{pump}/t).$$

Finally, defining the term that represents the volume of the pump divided by the time, (V_{pump}/t), for the pump as the pumping speed, s, and recognizing that the term (V_{pump}/t) is also the volumetric flow rate for the pump, a general expression for vacuum pump throughput becomes:

$$Q = N/t = (P/RT)\, s.$$

This model states that the number of molecules per unit of time that pass through the pump is equal to the (P/RT) multiplied by the pumping speed. If the situation for your vacuum chamber is such that the temperature is constant then R, the Ideal Gas Law constant, and T can be grouped with N/t to give:

$$(N/t)(RT) = (P)s$$
$$(Q)\,(RT) = (P)s$$

Thus, the ideal gas throughput,Q, for a vacuum pump as related to a specific gas temperature, 0^{o}C, is defined as the product of the pressure of the molecules entering the pump and the pumping speed of the pump in your vacuum system.

In practice, the throughput calculation uses two convenient short cuts. First, the value for RT is not calculated as long as the throughput value is always related to 0^{o} C. Second, the numerical value for pressure used in the throughput equation is the pressure at the entrance of the pump, i.e. the pressure in the vacuum chamber. With these shortcuts in mind, the throughput equations becomes:

$$Q \text{ (at } 0^{o}\text{C)} = (P)s.$$

It is important to remember that the product of pressure, P, with pumping speed, s, only provides a numerical value that is proportional to the actual number of molecules that pass through the pump at any instant of time. However, it will still be true that a vacuum pump with large throughput value will pull more molecules out of a vacuum chamber per unit time than a vacuum pump with a lower throughput value.

AP Student Homework Problem V(A)-2. Pressure Prediction

Using the Ideal Gas Law from the kinetic theory develop a model which predicts the pressure in the vacuum chamber after the pump has been running a specific amount of time.

Answer:

As long as the pump is on, molecules from the vacuum chamber are pulled into the pump and then pushed out into the surrounding outside the chamber. Thus, the pressure in the vacuum chamber will decrease as the number of molecules in the chamber continues to decrease. It is straightforward to relate the number of molecules in the vacuum chamber at any instant of time to the volume of the chamber and the pressure in the chamber at the same instant of time.

$$N_{chamber} = (PV_{chamber}/RT) = [(P/RT)(V_{chamber})]$$

Where P, T, and $V_{chamber}$ correspond to the pressure, temperature and volume inside the vacuum chamber. It is also easy to use the gas law to express the change in the number of molecules inside the pump, $d(N_{pump})/dt$, to the pump's volumetric flow rate, i.e. pumping speed:

$$d(N_{pump})/dt = (P/RT)(V_{pump}/t) = (P/RT)(s) = \{(P/RT)s\}.$$

In this last equation the $d(N_{pump})/dt$ term on the left is a symbol that just transmits the idea that it is the change in the value of the expression inside the parenthesis, N_{pump}, not the actual value of that expression that is of interest. The V_{pump} value divided by time is the definition of pumping speed

Unfortunately, these two expressions can not be equated since the change in the number of molecules in the pump at any instant of time, the second expression, is never equal to the number of molecules in the vacuum chamber, the first expression, at that same instant of time. However, if we pretend (assume) that, for any instant of time the pump is on,

(a) molecules only leave the vacuum chamber by going through the pump, and,
(b) no molecules enter the vacuum chamber while the pump is running

then the change in the number of molecules in the vacuum chamber equals the change in the number molecules entering the pump. Mathematically, this is a differential equation that can be stated as:

$$d(N_{chamber})/dt = d(N_{pump})/dt.$$

Simple substitution with the appropriate equation above will produce:

$$d([(P/RT)(V_{chamber})]/dt = \{(P/RT)s\}$$

moving the constant term, the RT term, and the chamber volume term on the left outside the differential symbol, the differential equation becomes

$$(V_{chamber}/RT)\ d(P)/dt = (s/RT)\ (P)$$

after canceling RT terms, moving the $V_{chamber}$ to the right, and grouping the pressure variables on the left the differential equation becomes:

$$(1/P)dP = (s/V_{chamber})dt.$$

Please note that the $(s/V_{chamber})$ term on the right involves the volume of the pump, as part of the 's" variable in the numerator, as well as the volume of the chamber in the denominator. These values are not equal since the pump volume in terms of the pumping speed, s, is much smaller than the chamber volume, V.

The solution to this differential equation model involves an integration of the differential terms on both sides of the equation. Fortunately this is easy to do. The integrals in question are found in any AP math textbook or first year calculus book. After integration the (1/P)dP term becomes:

$$\mathrm{Ln}\ (P/P_o)$$

while the $(s/V_{chamber})$dt term changes to

$$(s/V_{chamber})(t - t_o).$$

Thus, the differential equation can now be expressed as an integral equation by substitution of the appropriate terms and the model for the problem statement now becomes

$$\mathrm{Ln}\ (P/P_o) = (s/V_{chamber})(t - t_o).$$

Shorting the $V_{chamber}$ term to V, the antilog of both sides of the equation is taken and the general model becomes:

$$P = (P_o)\ e^{[-(s/V)(t\ -\ t_0)]}.$$

If this model is applied to a vacuum system with an initial pressure, P_o,equal to 1 atmosphere and the starting time for the vacuum pump, t_o, is designated as 0 minutes, then the model becomes:

$$P = (P_o)\ e^{-[(s/V)(t)]}$$

Thus, the value of P, the pressure inside the vacuum chamber, is now a function of the time the pump is operating. Actual values for pressure inside the vacuum chamber as a function of pump operation time can be calculated once the volume of the chamber,V, and the pumping speed, s, are known.

AP Student Homework Problem V(A)-3. Percent Pressure Drop

Use the general model that related the pressure in a vacuum chamber to the time in the vacuum pump to derive an equation that will express this decrease in pressure as a percent of the starting pressure value.

Answer:

The general model for pressure in a vacuum chamber with starting pressure P_0 is:

$$P = (P_o)\, e^{[-(S/V)(t - t_0)]} .$$

In this equation the V term is the printing space saving version for the volume of the pump, V_{chamber}. If the model always starts when the pump is turned on, $t_o = 0$, the expression becomes

$$P = (P_o)\, e^{-[(S/V)(t)]}$$

and the equation that expresses pressure in the vacuum chamber as a percent of the starting pressure becomes,

$$P/P_o = e^{-[(S/V)(t)]}\, x100$$

where the 100 is used to get a % answer instead of a decimal value.

V(B). Modeling Vacuum Pumpdown Characteristics

When designing a vacuum system, one of the most basic considerations is the time required to evacuate a vessel to a given pressure. This time typically ranges from a few minutes to hours but can, on occasion take a few days. If the pump is assumed to be operating at constant pumping speed and efficiency then the pressure vs. time behavior exhibited by a system is given by the following model:

$$P = P_o \; e^{-[(s/V)(t)]}$$

where P_o is the initial pressure at zero time t, P is the instantaneous pressure at any time t greater than zero, V is the volume of the chamber, e.g. in liters, and *s* is the pumping speed of the pump in liters/second. The quantity V/*s* is known as the time constant of the system and can be defined as the time it takes to pump the system to a pressure of P_o/e or 37% of its initial value.

The pumping speed of the pump and the time constant of the vacuum system can be obtained from pressure vs. time characteristics during the pumpdown cycle as illustrated in Figure 20. There are three main objectives of the following exercise: to experimentally determine the pumping speed of the pump and the time constant of the vacuum system, and to use these values to understand some of the problems inherent in the design of vacuum systems.

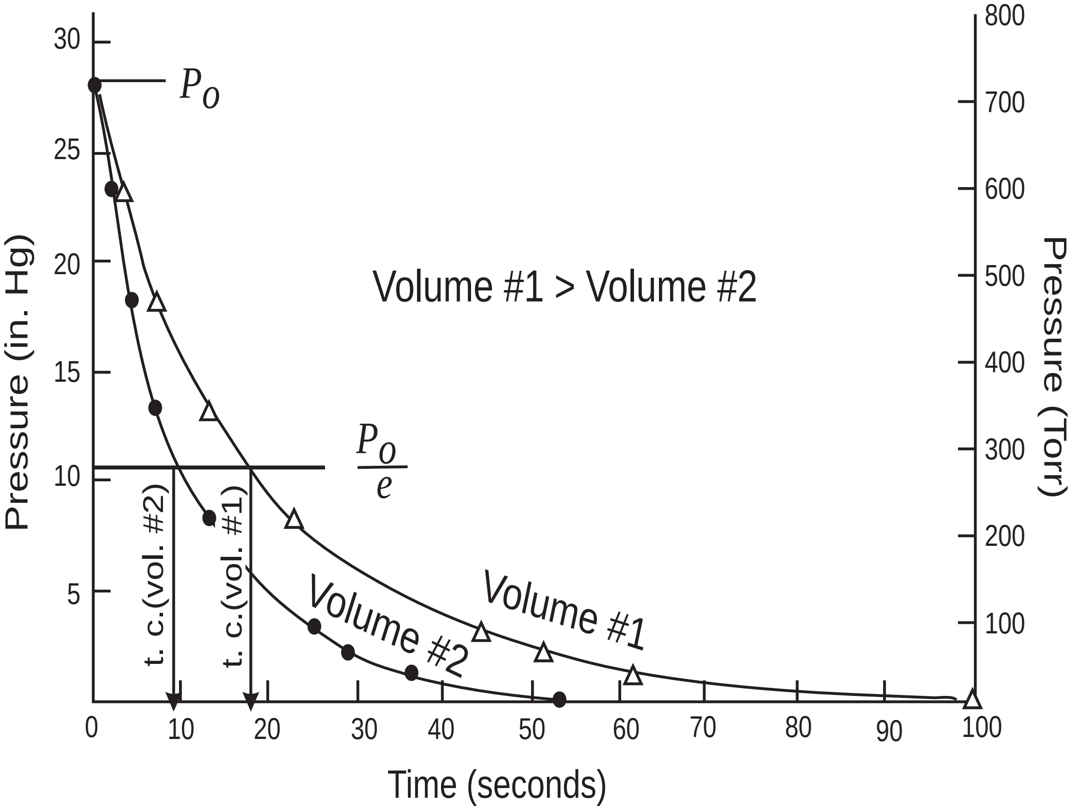


Figure 20. Pumpdown characteristics expressed as time - pressure curves for two different volumes using a positive displacement mechanical vacuum pump.

Experiment #1. Pumpdown Exponential Model Verification

Measure the volume of the vacuum chamber and record the vacuum gauge value for atmospheric pressure. Start a pumpdown cycle for the properly assembled vacuum system by switching on the pump. Record the pressure every two or three seconds. Record data until the pressure is at the lower limit of the gauge. Allow the vacuum chamber to return to atmospheric pressure and repeat the pumpdown process again. Record the data for this second pumpdown as a separate set of data. Repeat the experiment yet one more if your data is not consistent in time.

1. Plot the pressure and time (in seconds) data pairs for the average of your experimental runs on the graph paper provided. Answer the following questions.
   - (a) What is the value on the time axis when the pressure has fallen to 37% of its original value?
   - (b) What is the value on the time axis when the pressure has fallen to 14% of its original value?
   - (c) Does the pressure ever go to zero?

2. Plot ln(pressure) vs. time for the same data on a different graph sheet, as illustrated in Figure 21.

If the vacuum system is following the ideal gas law model and if you have not made any errors in your data collection and plotting activity, the ln(P) vs. time plot should be a straight line, following this relationship:

$$lnP = -(s/v)t + lnP_o$$

- (a) What is the algebraic significance of the slope of your linear plot?
- (b) What is the algebraic significance of the pressure axis intercept of your linear plot?
- (c) What extra information is needed to determine the pumping speed, s, of the pump used in your experiment?

3. What is the pumping speed of the system?

4. Change the volume of your vacuum system.
   - (a) Plot the pressure readings vs. time in seconds for this new volume vacuum chamber on the same graph used in part 1 above.
   - (b) Plot the ln(P) vs. time data on the same graph used for part 2 above.
   - (c) What are the values on the time axis when the pressure has reached 37% and 14% of its original value?
   - (d) What is the pumping speed for this new vacuum chamber? (Is it going to be a different value than the pumping speed calculated above in part 3?)

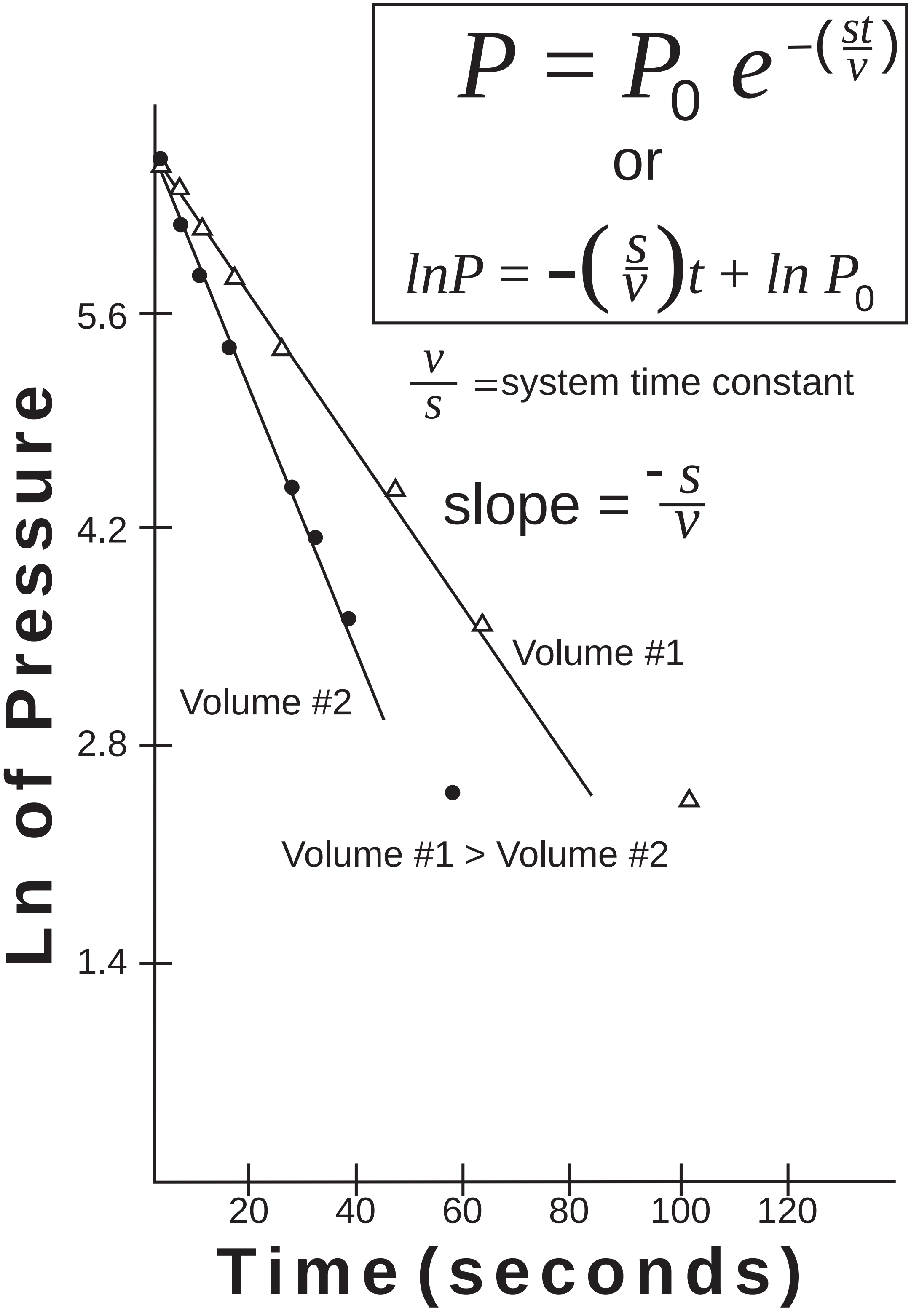


Figure 21. Pumpdown characteristics expressed as time - Ln pressure curves for two different volumes pumped with a positive displacement mechanical pump.

Data Sheet - Pumping Speed

Volume of vacuum jar ___ = ______________

Time	Pressure	Ln Pressure

V(C). Theory of Pumpdown Characteristics

The theory behind pumping speed is based on maintaining a mass balance in a vacuum chamber. Making a mass balance in the vacuum chamber:

$$[Accumulation] = [Input] - [Output]$$

which can be written as: $\frac{dN}{dt} = -\dot{N}$ (1)

where N is the number of moles in the vacuum chamber, at any time, and $\dot{N}$ is the rate at which moles are leaving the chamber. Assuming ideal gas behavior:

$$N = \frac{PV}{RT} \qquad \dot{N} = \frac{Ps}{RT}$$

where P is the absolute pressure, T is the absolute temperature, R is the ideal gas constant, V is the chamber volume and s is the pumping speed.

Replacing in equation 1:

$$\frac{V}{RT}\frac{dP}{dt} = -\frac{Ps}{RT}$$

then at constant temperature and chamber volume:

$$\frac{dP}{P} = -\left(\frac{s}{V}\right)dt$$

Integrating over the initial pressure *(P_o)* at time= 0 to the final pressure (*P)* at time *t* and assuming an effective pumping speed s_{eff}

$$\ln\left(\frac{P}{P_o}\right) = -\left(\frac{s_{eff}}{V}\right)t$$

or:

$$P = P_o e^{-\left(\frac{s_{eff}}{V}\right)t}$$

V(D). Models and the Real World

Operational Range for Real Vacuum Pumps

There are three broad categories of vacuum pumps: positive displacement pumps, momentum transfer pumps and capture pumps. The type of pump used for a particular application depends on the required pumping speed, the gas load to be pumped, the nature of the gases to be pumped, and the ultimate pressure to be achieved. Figure 24 shows the operation ranges for a number of different types of pumps. In this workshop you will be using a an oil sealed, positive displacement, mechanical pump, which according to Figure 24, has a pumping limit of about 0.001 Torr.

As Figure 22 suggests, there is not one type of pump that can reach the entire pressure range. For example, if the pressure in the chamber is to approach the low values found in outerspace, i.e. < $1x10^{-12}$ Torr, more than one type of pump is required. Pressures as low as nanotorrs or picotorrs are possible in a vacuum chamber when a sublimation pump is attached to that vacuum chamber. Unfortunately, a sublimation pump does not work very well, if at all, when the pressure in the chamber is one atmosphere. Thus, a different pump or set of pumps must be used to bring the pressure in the vacuum chamber down to 0.001 millitorr before the sublimation pump can be turned on. It is this limited property of vacuum pumps that makes vacuum science and technology so interesting, challenging and expensive.

As suggested by the experiments and homework exercises presented so far, the important parameters associated with a vacuum pump are its throughput and its pumping speed. These two characteristics will determine the vacuum chamber's ultimate pressure. Naturally, this best, lowest, pressure in the chamber depends on the operating range of the pump being used. It is not reasonable to expect the best low pressure, the ultimate pressure, for an oil-free mechanical vacuum pump to be one nanotorr. However, different companies make different types of oil-free mechanical pumps and the performance of all of these pumps varies with its quality and construction. Thus, a pump's throughput, pumping speed and, of course, cost help with the sometimes difficult decision of which pump to buy for a specific vacuum chamber application

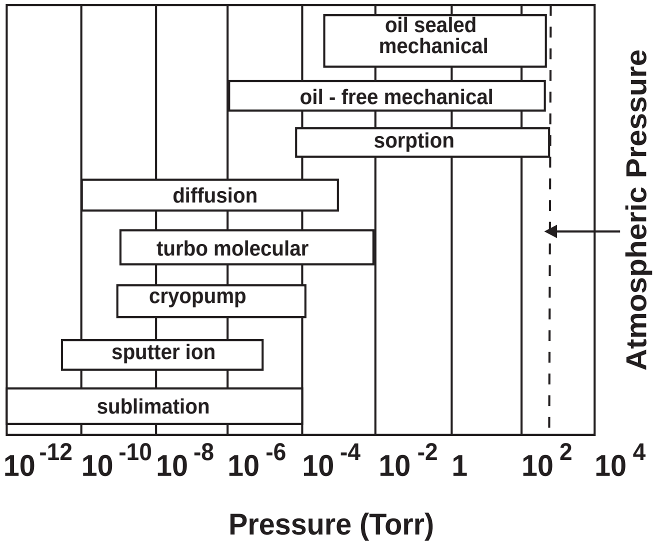


Figure 22. Operational ranges for various vacuum pumps.

V(E). Building a Model from Data

Another common procedure for building a model is to conduct an experiment that explores the scientific or engineering effect of interest and then collect the data that describes the observations generated from the experiment. This data can then be examined mathematically to obtain a functional relationship that has the same characteristics as the graphed data. The procedure is empirical but effective and useful. There are two general approaches. The first is to plot different functions and compare them to the experimental data. If a functional form is identified that matches the form of the experimental data, the coefficients associated with the function are altered to bring the function into coincidence with the data plot. The other way is to determine a statistical least squares plot that best matches all of the experimental data. The better of these two approaches is the one to use.

Experiment #2: Weight, Force and Pressure Correlation

Shown in Figure 23 is a plot of the force of atmospheric pressure versus cross-sectional area under vacuum. The model to create the curve in Figure 25 is based on the standard value for atmospheric pressure at sea level, 101,000 Pascals, and the curve is generated with increasing cross-sectional areas according to:

FORCE = pressure x area

This activity will not be performed in this workshop but can be conducted in the classroom as a demonstration or an experiment. Vacuum plates, purchased from a scientific teaching supply vendor or constructed (see Appendix I), or simple vacuum suction cups used to hang household items to a wall can be used. As shown in Figure 24, a fish scale could be used with the suction cups to measure the force of the atmosphere.

The objective for students is to construct their own force versus area curves with different cross-sectional areas. Their curve based on actual measurements should follow a predictable pattern similar to the model curve they construct.

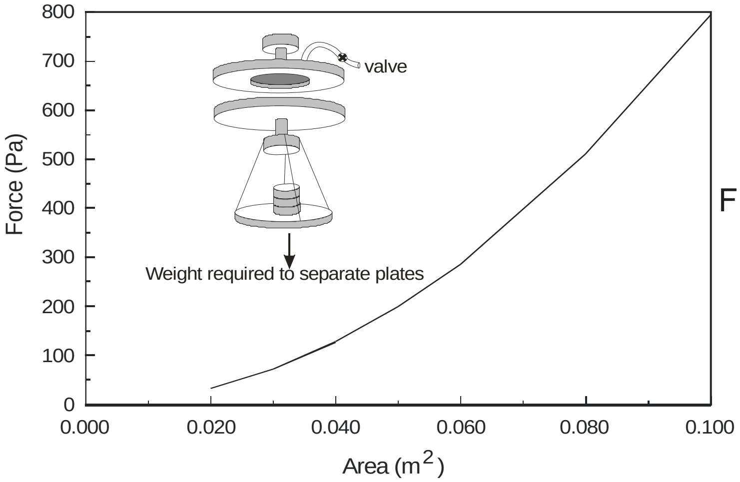


Figure 23. A model plot of the force of atmospheric pressure versus cross-sectional areas of vacuum vessels.

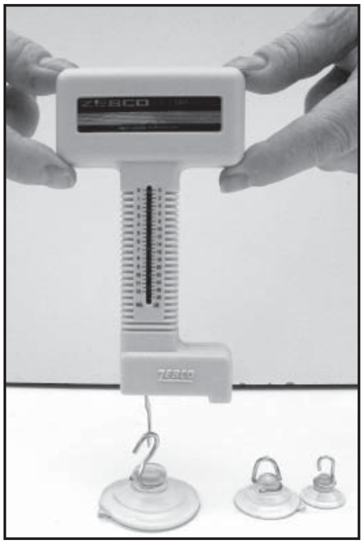


Figure 24. Measuring the force of atmospheric pressure on a household suction cup with a fish scale.

Experiment #3: Altitude to Pressure Correlation

This activity is an extension of Experiment #2 with the added opportunity for students to correlate pressure measurements in a vacuum chamber to similar pressure values at fixed altitudes above sea level. This experiment requires the use of an altimeter capable of withstanding the very low pressure environment of the vacuum chamber. In addition to this valuable learning experience, the experiment provides an opportunity to introduce to the students the idea of using an experimental apparatus to simulate a much more complicated environment. This general idea actually might be considered to be even yet another type of model, known as a physical model, where a small scale and idealized system is used to represent and take data from a larger, more complicated and real situation or environment. Granted, this specific example is a simple experiment for a simple simulation but this quality should help make the point easier for the students to appreciate. Typical altimeter readings taken in a single stage pumped vacuum system are illustrated in Figure 25.

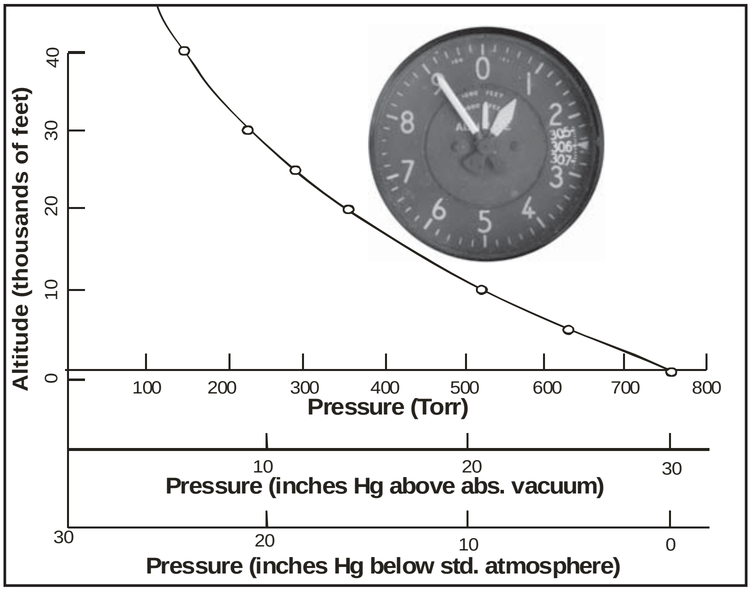


Figure 25. A plot of altitude versus pressure obtained from an altimeter placed in a vacuum chamber.

VI. GAS PHASE MODELS

VI(A). Dealing with the Ideal Gas Model

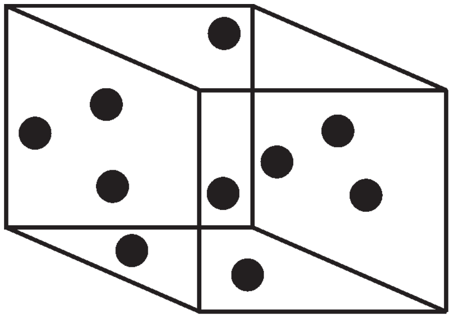
An ideal gas is a concept defined as one in which all the molecules are spheres that do not react or interact with each other and all collisions are elastic. This is illustrated in Figure 26. Observations of the behavior of gases by Avogadro, Boyle, and Charles, as illustrated in Figure 27, led to the Ideal Gas Law in 1834 by Emil Clapeyron. The Ideal Gas Law is expressed as:

$$PV = NRT$$

Where P = system pressure, V = system volume, T= system temperature, N = number of moles in the system, R = universal gas constant or an experimentally determined factor that makes the units of both sides of the equation consistent. The various units for P, V, n, and T are listed in Table 4. If we imagine an ideal gas moving though a pipe, the ideal gas law could be written to reflect the idea that a volume of gas would sweep through the pipe in a measured amount of time. This volume divided by time idea, V/t, would be the speed of the gas through the pipe. The equation for this idea would be expressed as:

$$P(V/t) = NRT$$

and we have used this idea of a volume of gas moving for a period of time in section V.



ASSUMPTIONS

1) molecules in the container are spheres that do not react or interact with each other

2) all collisions are elastic

Figure 26. The Ideal Gas Model

1. Relationship of pressure to the number of molecules

Avogadro’s Observations: Pressure is directly proportional to

a) the number of molecules

$$P \propto n$$

b) the temperature of the system

$$P \propto T$$

2. Relationships of volume, pressure and temperature

a) Boyle’s Law

$$P_1V_1 = P_2V_2 = \text{constant}$$

b) Charles’s Law

$$V_1/T_1 = V_2/T_2 = \text{constant}$$

Figure 27. Observations that led to the Ideal Gas Law

Table 4. Units for system pressure, volume, temperature and mole concentration for an ideal gas

System Pressure	System Volume	number of molecules in the system	System Temperature
atmospheres	liters	moles	Kelvin
mm of Hg	cubic meters	gross	degrees Celcius
Torr	quarts	dozens	degrees Fahrenheit
others	others	others	others

VI(B). Boyle's Law

For a system composed of a fixed quantity of an ideal gas at constant temperature, it is observed that the measured system pressure is inversely proportional to the volume occupied by the gas. This relationship is known as Boyle's Law and is named after the physicist Robert Boyle (1627-1691) who first stated it. It may also be written as follows: if the temperature of a confined gas does not change, the product of the pressure and the volume is constant. This relationship is illustrated mathematically in Figures 27 and 28.

Boyle's Law can be observed in any closed isothermal system in which the pressure or volume can be varied. A model for Boyle's Law can be developed providing that these quantities can be measured. A good experimental setup for obtaining the pressure vs. volume data needed might be a closed vessel of known initial volume with a plunger at one end and a pressure gauge at the other. The pressure is read directly whereas the volume is proportional to the height of the plunger. If the cylinder is calibrated, then volume tic marks can be used.

Another useful but somewhat more challenging system is one demonstrated earlier in this workshop - the expanding balloon. The balloon represents a closed system whose volume and pressure is changed by varying the force of the surrounding air on its exterior surface. (See Demonstration #1: Expanding Balloon.) If an expanding balloon is to be used to observe Boyle's Law, there are two initial goals to the exercise. The first objective is to obtain pressure - volume data for a balloon at reduced pressures in a vacuum system. The second is to observe and characterize the difference between a model and experimental observations.

Experiment #4: Boyle's Law: Observation and Model

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1. Determine the initial volume of the balloon.

2. Place the balloon in the vacuum chamber and begin the pumpdown. Since the balloon is expanding to equalize its internal pressure with the reduced external pressure of the vacuum jar, the pressure can be read directly from the gauge. Note that you can stop the pump for brief periods of time to take data points or you can pump until the balloon reaches maximum expansion, shut off the pump, and vent the system in controlled increments of pressure, taking pressure and volume readings as the pressure increases. (Be sure to take enough readings to develop the shape of the V vs. P plot.)

3. The only other measurement that is needed is that of the volume of the now nearly spherical expanding balloon in the vacuum jar. There are a number of ways to determine the volume. We will leave the approach to making this measurement up to you. Be creative but remember to be as accurate as you can!

4. Plot the experimental volume - pressure data pairs as volume vs. pressure, on the graph paper provided and draw a smooth curve to represent the data points.

5. Using the initial (atmospheric) PV product where the volume of the balloon is most accurately known, calculate a theoretical volume for each recorded pressure and plot this PV data on the same graph. This is the model curve.

6. Compare the curves from the experimental results with the theoretical calculations. Discuss the likely sources of error. Aside from measurement errors how does the choice of the experimental setup play a role in the inaccuracies present in the data? (Hint: How does the balloon itself affect the volume measurements?).

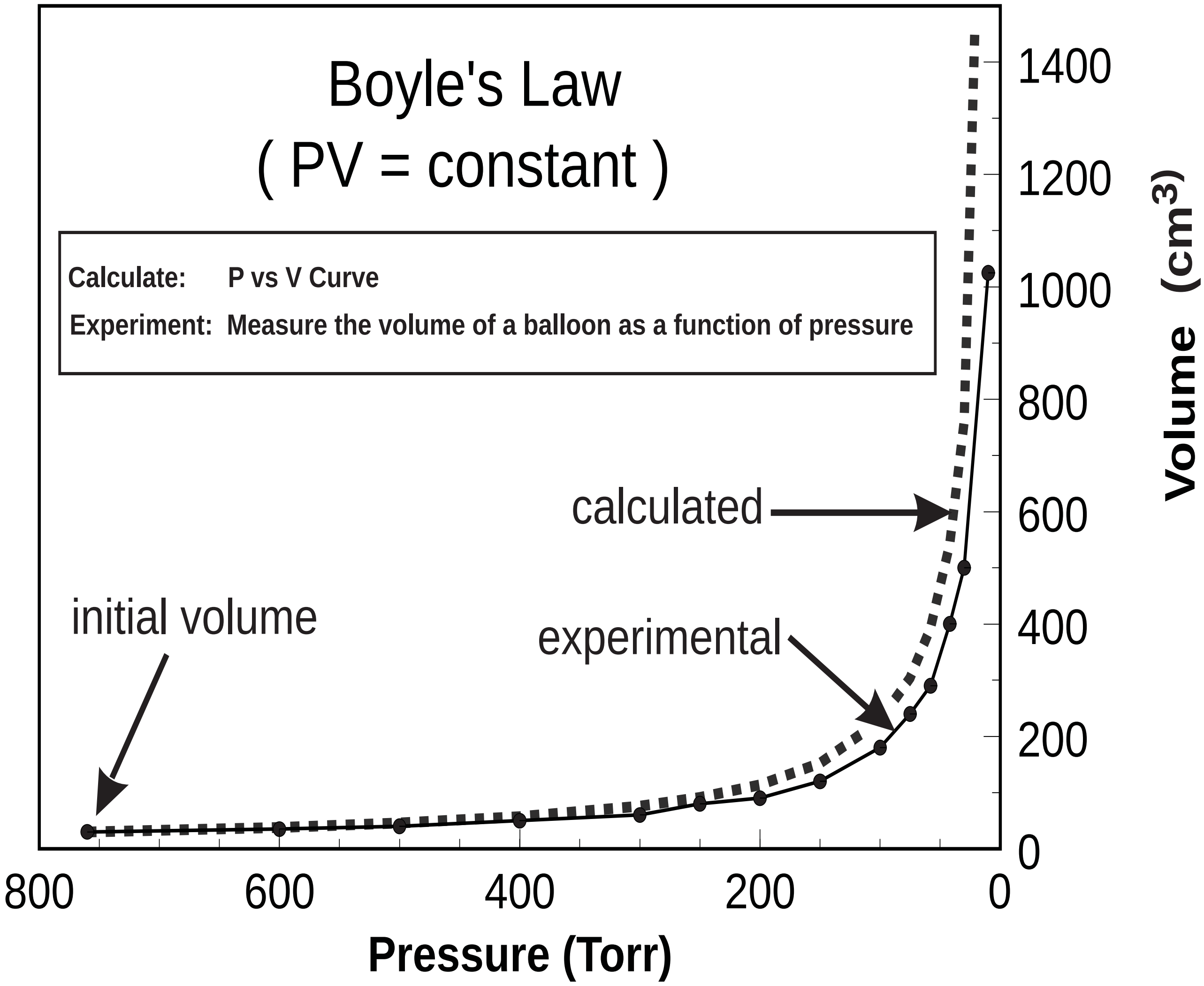


Figure 28 Experimental and model curves of volume versus pressure for a balloon in a vacuum.

Data Sheet - Boyle's Law: Observation and Model

Initial volume of balloon =

Pressure	Measured Volume	Theoretical Volume

VI(C). Polytropic Process

An great number of situations of interest to chemical and mechanical engineers deal with the movement of a fluid. Specifically, the engineer is focused on any gain or lose of energy a fluid experiences as it is transferred from one situation to another. Certainly, one reason for some energy lose as the fluid moves is do to friction. However, from any application perspective, the engineer's interest is related to fluid energy changes in the form of heat or work.

The manipulation of a fluid, so that energy is expressed as heat or work, is easily done by devices within things that vary the temperature, pressure, or volume of the fluid as it moves through a particular device within that thing. As might be imagined, there are just as many variations of these three variables to generate energy changes in a moving fluid as there are different types of things from shuttle boosters to steam locomotives that use these energy changes to function properly. How can anyone keep track of all of these various possibilities?

The trick is to define a vocabulary that is easy to use but allows anyone to clearly define the specific changes in the fluid's temperature, pressure, and volume of interest and also clearly indicate the amount of heat or work that resulted from those changes. Some example processes that use words in this heat/work vocabulary are isobaric process, isotropic process and isothermal process. Although one way to generalize these processes is to call them a "Something Tropic" process, the physics and engineering community has elected to use the phrase polytropic process.

A polytropic process is a generalized process that can be modeled such that the pressure times the volume raised to a power always equals a constant value, PV^n = constant. (This looks very similar to one statement of Boyle's Law, PV = constant.) Figure 29 on the following page illustrates three example curves of three common polytropic processes. The curves are distinctive and easily identified by the specific values of k as shown. Isobaric (constant pressure) processes are represented by the horizontal line that results when n = 0. This type of process occurs whenever a fluid passes through a device such that its temperature or volume (or both) change, while the pressure of the fluid remains the same as it was before it entered the device. Processes occurring on the earth's surface while exposed to the atmosphere are considered to be isobaric for most practical purposes.

The other plots shown in Figure 29 are two other examples of polytropic processes: an isothermal process (Boyle's law describes an isothermal process when n = 1) and an isotropic process (where n is a number other than 0 or 1). Isothermal processes are processes that occur because a fluid passes through a device such that the temperature remains constant while the pressure and/or volume change. An isotropic process is a process that occurs because a fluid passes through a device that changes the fluid's pressure, temperature and/or volume but does not change the general order (or disorder) of the fluid, i.e., there is no phase change in the moving fluid.

At this point, these terms might be new to you. To find an anchor for this expanding vocabulary please note that;

1. A process that follows Boyle's law is an isothermal process.
2. The isothermal process (n = 1) is a specific example of a polytroic process.

In summary, a polytropic process is a process that manipulates the temperature, pressure or volume of a fluid so that there is a predictable energy change as the fluid passes through a device in that process.

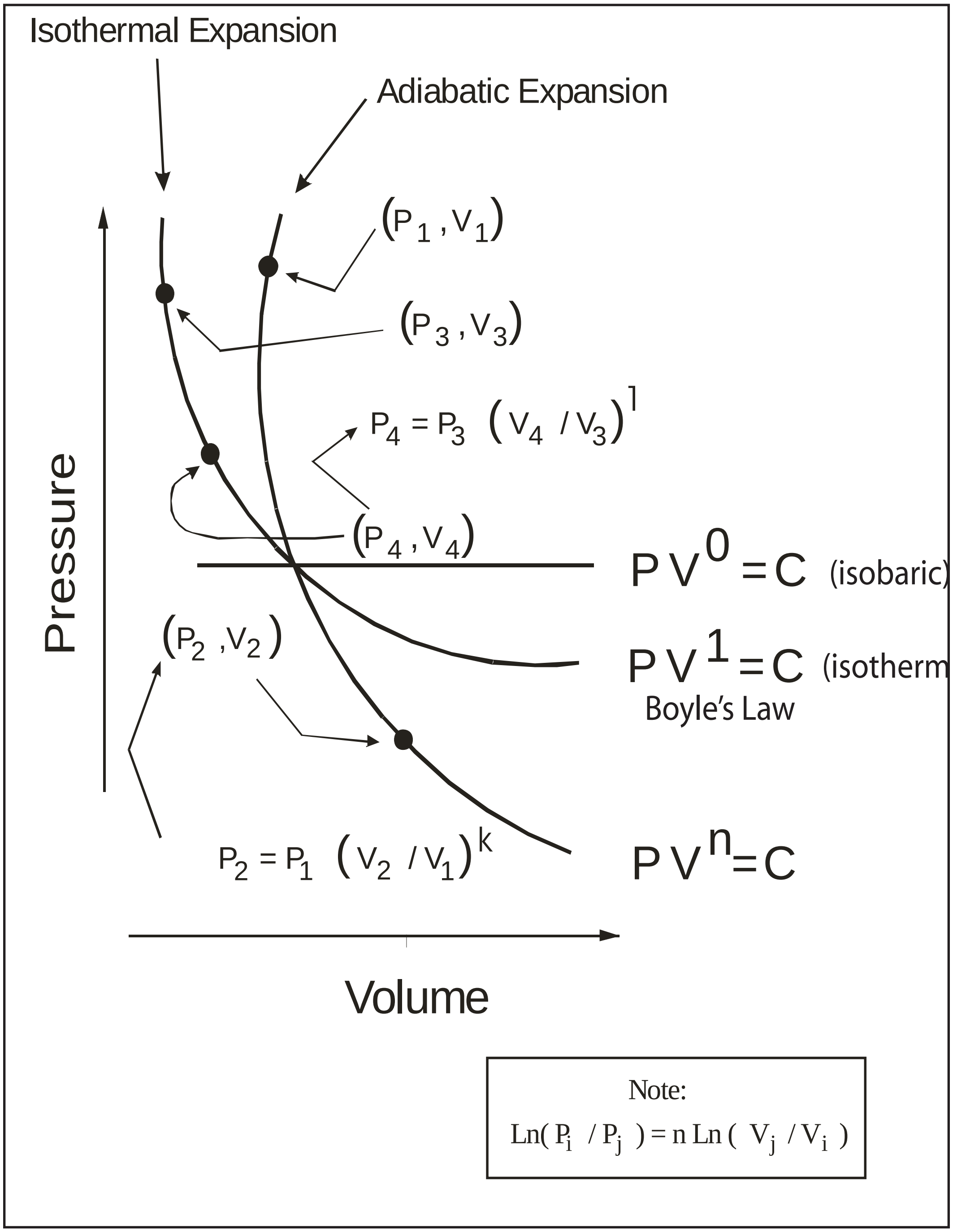


Figure 29. The range of polytropic processes

VI(D). Polytropic Perspective

Why not just use Boyle's and/or Charles' Law to describe pressure, volume and temperature relationships for gases? These are quick questions that come to mind, especially if polytropic sounds Greek to you. However, Boyle's and Charles' Laws are just specific cases of polytropic processes that apply to ideal gases and are very useful for gases that behave that way. Unfortunately, most gases in real applications do not behave as ideal gases. Therefore, the aim is to develop an understanding of gas behavior since any combination of changes of P, V, and/or T require a polytropic perspective.

This alternate way, the polytropic model, to describe P, V, T relationships was developed from applications that involved getting usable work from a moving fluid i.e., applied thermodynamics. Engineers and applied scientists have found polytropic models to be more useful when describing many applications such as steam engines and refrigeration cycles because these models are not restricted by the assumptions of ideal gases. Ideal gas assumptions are often inappropriate when dealing with supersaturated steam, pressurized coolants, and other fluids used in common engineering systems.

Additionally, for practical applications, it is more important to describe the behavior of the gases correctly (i.e., model their behavior) under their operating conditions rather than to know how they might behave if they were assumed to be ideal. The variable exponent k, in the polytropic equation allows such flexibility for dealing with ideal gases. Ultimately, the polytropic model allows for more accurate determination of energy requirements or production in terms of work or heat for engineered systems. Needless to say, accuracy determining the energy consumed or produced in a real operating process is very important. This is the case if all power production is used to produce electricity and/or heat for public consumption, or is converted to mechanical work to run factories, automobiles, etc.

Experiment #5: *Polytropic Exponent Determination*

1. Collect a set of P-V data points with temperature held constant. (You may use the data from Experiment #4, or collect a new set of P-V data, so that you have 2 replicates for the same experimental measurements.)

2. Determine the value of “n” for each experimental data set that you have. (Consider using the average n if you have more than one P-V data set.)

3. Plot your P-V data set corrected for polytropic behavior as illustrated in Figure 30.

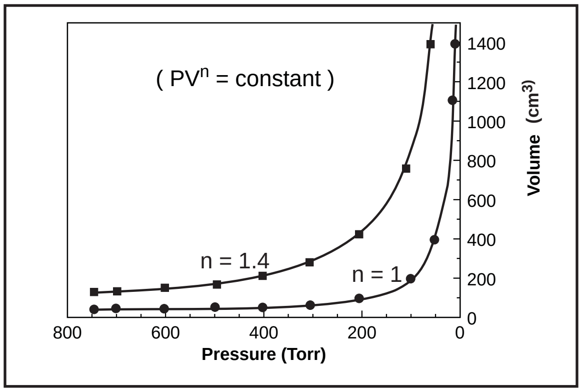


Figure 30. Polytropic form of Boyle's Law

Data Sheet - Determination of Polytropic Exponent

Initial (PV) Product =
Initial volume of balloon =

Pressure	Measured Volume	Pressure Volume

VII. MULTIPHASE BEHAVIOR

VII(A). Latent Heat of Vaporization

When a liquid is boiling, molecules constantly leave the surface and the liquid changes to the gas state. As each molecule leaves the surface it will take some thermal energy with it. The amount of thermal energy, heat, will be determined by the heat required to reach the boiling point of the liquid plus the latent heat, hidden heat, of vaporization defined as “the quantity of heat required to change unit mass of liquid into vapor at the boiling point”. This processes is illustrated in Figure 31 by the plateaus in the time-temperature curve between phase changes.

Normally, the thermal energy necessary to reach and sustain boiling is supplied to the liquid from an external source. However, if the liquid is made to boil by reducing the pressure over the surface with no external heat provided, the process is reversed. The molecules leaving the liquid surface and entering the gas state still require their latent heat as before; but, since no heat is being supplied, the thermal energy is obtained from the molecules that remain in the liquid state. This will result in a reduction in the kinetic energy of the remaining molecules of liquid. Their loss of kinetic energy will cause a drop in the temperature of the remaining liquid.

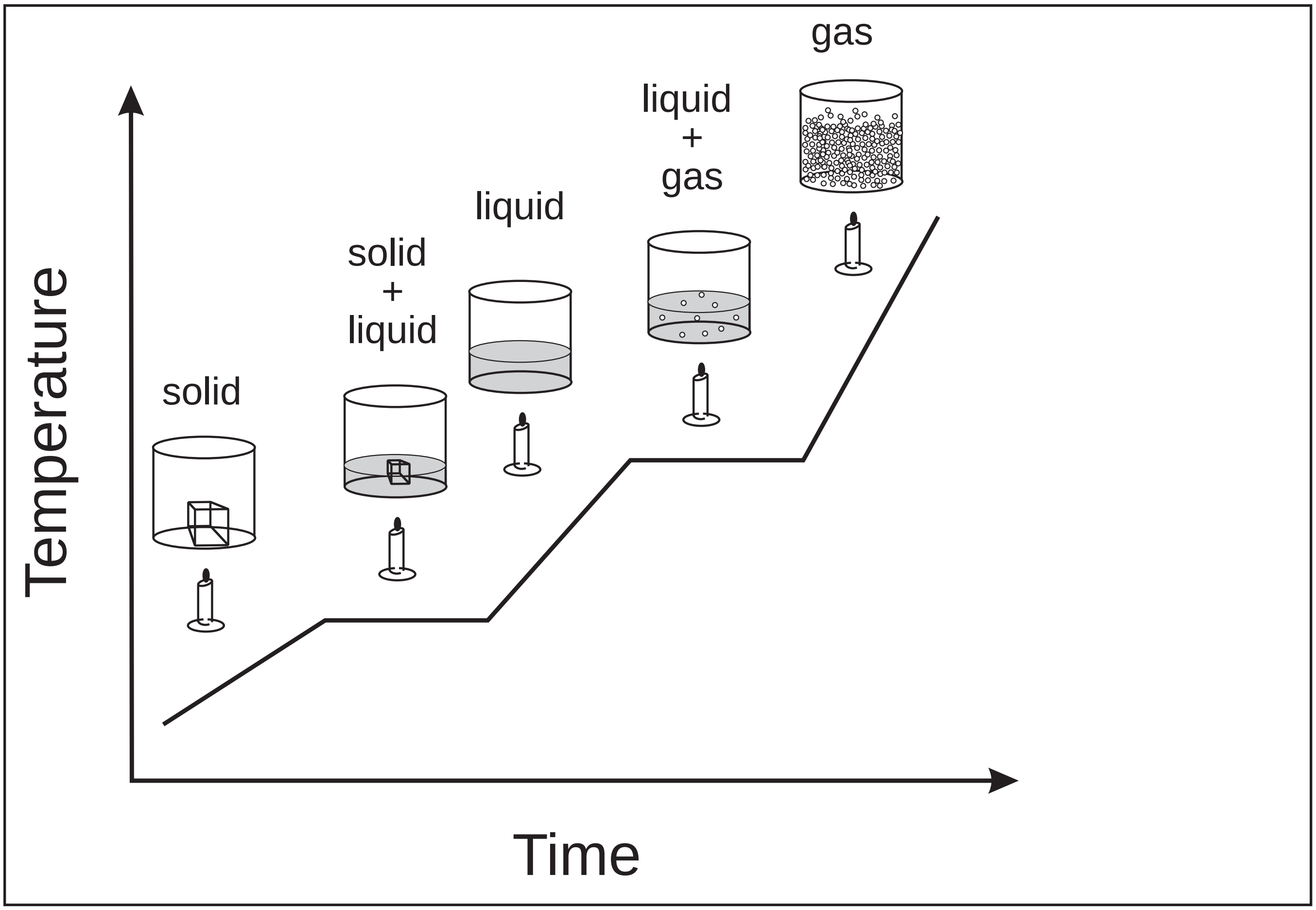


Figure 31. Temperature observations during phase changes of water.

If enough cooling occurs, the liquid will undergo yet another phase change to a solid. As shown in Figure 30, when all three phases are present and in equilibrium, the temperature and pressure conditions at which this occurs is called the “triple point”. Once the triple-point is reached, additional pumping will remove the gas phase and the solid phase (ice in the case of water) will remain. As shown in Figure 32, the triple point of water in the absence of air occurs at a temperature of 0.16^{0}C and a pressure of 4.58 mm Hg. In the presence of air at one atmosphere, the three phases are in equilibrium at 0^{0}C. However, since the partial pressure of water vapor is only 4.58 mm Hg, the triple point is lowered because both the solubility of air in liquid water and the increase in pressure from 4.58 mm to 760 mm lower the freezing point.

Demonstration #4: Latent Heat of Vaporization

The classic way to demonstrate latent heat is to heat a substance such as water while observing the temperature. If the water begins in the liquid phase in a one atmosphere environment, the temperature rises with the input of heat until the boiling point is reached. As long as the water boils, the heat energy being put into the system is used for the phase change and does not increase the kinetic energy of the molecules, i.e., the temperature of the water does not increase until the phase change from liquid to vapor is complete. To demonstrate latent heat of water in a vacuum chamber, the process is carried out in reverse, as discussed earlier in this section.

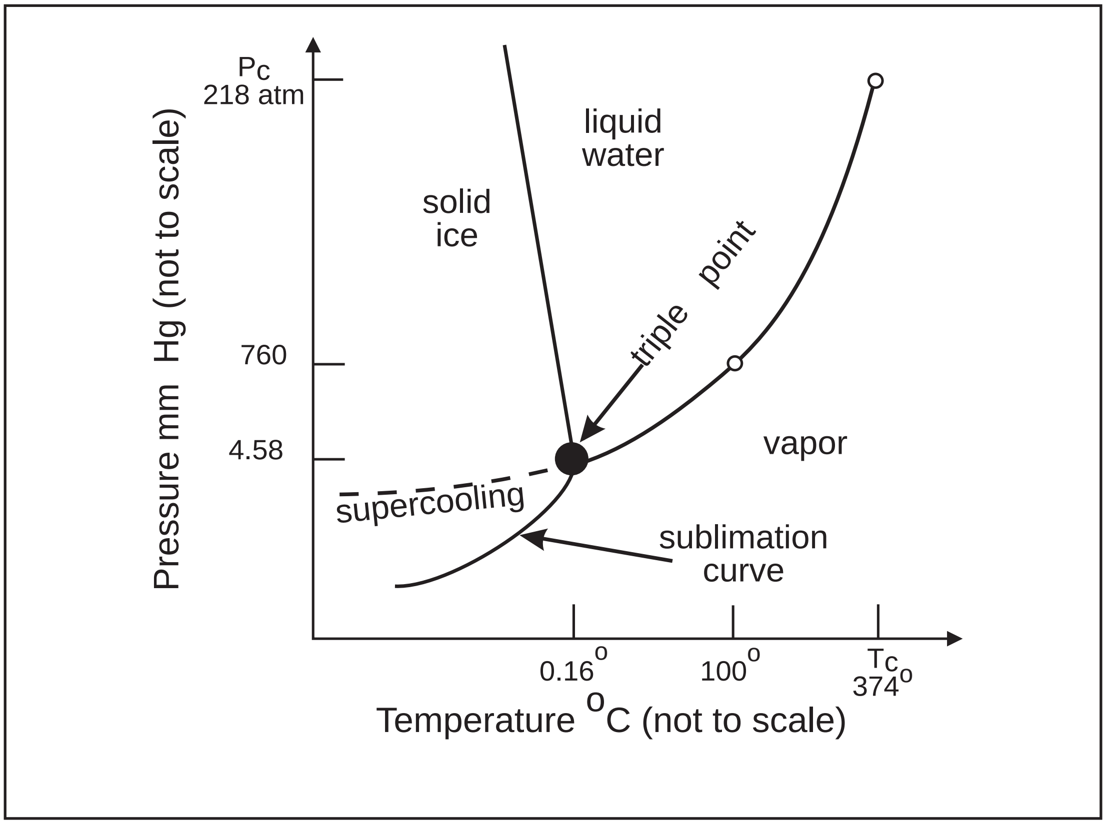


Figure 32. Phase diagram of water.

To demonstrate the effect, a test tube containing a small volume of water is placed in the vacuum enclosure, as shown in Figure 35. As the pressure in the vacuum enclosure is reduced, the air that is dissolved in the water will be released and observed as small bubbles. The release of this dissolved air is quickly followed by a more violent bubbling as the water begins to change from the liquid to the gas state. Figure 34 indicates, the pressure (the y axis values) temperature (the x axis values) combination needed to instigate this rapid evaporation. As the vacuum pump reduces the pressure above the liquid water, the temperature of the water decreases due to the evaporative cooling. If the pressure in the chamber continues to drop, the temperature will decrease to a value, 0.16 degree C, at which point the liquid water is changed to solid water, ice, and a large amount of heat, the latent heat of fusion, is released. At this point, some of the liquid water is changed to ice while the released latent heat forces some of it to rapidly evaporate. This temperature and pressure combination is called the triple-point because, at this stage of the vacuum pumping process, all three forms of water- solid, liquid, and gas coexist and we can observe a curious occurrence, i.e., as some of the liquid water changes to ice while some of the water changes to gas.

The success of this demonstration depends on the complete removal of water vapor from the vacuum chamber as soon as it formed and thus the need of a vacuum pump with sufficient pumping speed. However, the pump's pumping speed is drastically reduced when the pump oil and pump cavities become saturated with water. To avoid this saturated condition, it is necessary to use a very small amount of water in the test tube for this demonstration. To further insure that you have sufficient pumping speed, you can add an additional non-mechanical "pump" by placing a small open container of dried desiccant (such as calcium carbonate or calcium sulfate) next to the water container in the vacuum chamber. The desiccant acts as a non-mechanical water pump, or, in this case, a chemical pump, because it removes some of the water vapor from the vacuum system by trapping it in its chemical structure, preventing it from contaminating the pump oil. Technically, it provides additional pumping capacity to reach the “triple point.” You may wish to run the experiment with the desiccant first, then repeat it without the desiccant to demonstrate that the mechanical pump alone may not be able to reduce the pressure sufficiently to freeze the water.

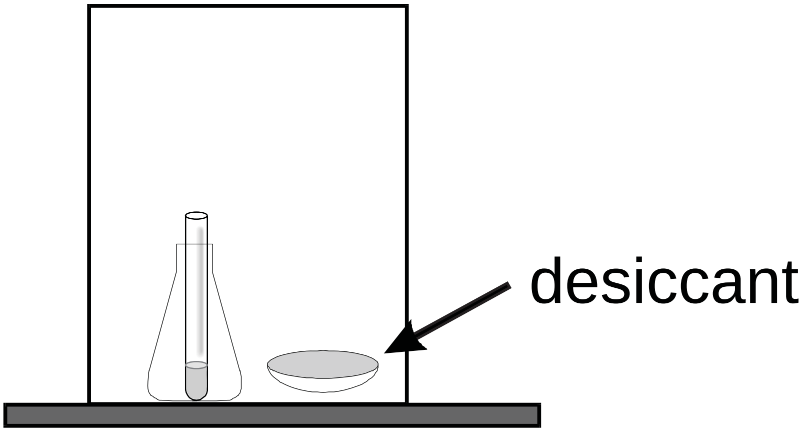


Figure 33. Setup for demonstration of the triple point of water in a vacuum chamber.

VIII. WORK & ENERGY ENGINEERING APPLICATIONS

VIII(A). Heat and Work as Energy Forms

Changes in the physical state of substances and movement of a fluid result in transfers of energy from one system to another. Two modes exist for such energy transfer. These are heat and work. Heat is an energy transfer mode that results in a change in temperature of a substance, while work is an energy transfer mode that results in some type of movement.

These two energy transfer modes should not be confused with forms of energy, such as chemical, electrical, mechanical, nuclear, etc. These forms of energy are different ways that molecules store and express their energy. These descriptive terms give users information about how energy is stored and kept ready to be released as heat or work to the outside environment.

Energy transfers expressed quantitatively as work (W) or heat (Q) occur as a substance changes from one thermodynamic state to another. A thermodynamic state is defined as a condition of the substance that can be defined by a unique set of values of the variables that describe it. The variables that describe a thermodynamic state include pressure, volume, temperature, internal and external energies, entropy, and others. For energy to be transferred as heat or work at least one of the variables describing the thermodynamic state must change, thus defining a new thermodynamic state. This process is illustrated in Figure 34, where the variables P, V, and T describe the amorphous regions that represent the two different thermodynamic states. Note that heat and work are not attributes of either thermodynamic state, but are the reason that a substance transfers between two different states.

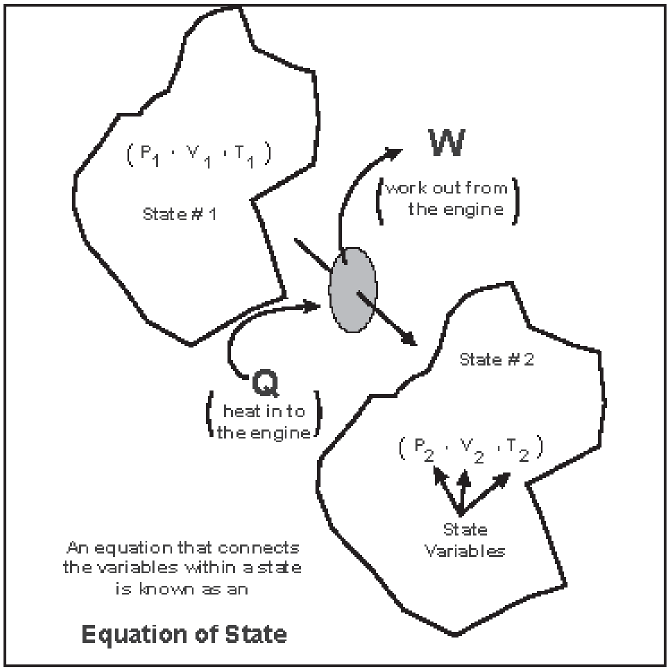


Figure 34. Illustration of two forms of energy transfer: work and heat.

VIII(B). Adiabatic Expansion

When a gas is suddenly expanded, it pushes molecules outward. This is described by saying that the expanding gas does work on its surroundings. This external work results in a loss of energy by the molecules which is reflected as a decrease in temperature. An adiabatic expansion is one in which no heat is allowed to enter or leave the gas. In this case, the decrease in temperature will be permanent. An adiabatic expansion can be achieved by thoroughly insulating the chamber in which the gas exists, as illustrated in Figure 35.

The cooling effect of an adiabatic expansion is demonstrated by mounting a thermometer inside the vacuum jar and turning the pump on. The molecules that remain in the chamber rapidly expansion and the temperature drops several degrees. This temperature drop is not maintained since heat radiated from the surrounding environment warms the remaining air up again.

A specialized application of the cooling produced by adiabatic expansion is the Cloud Chamber as perfected by C.T.R. Wilson in 1900. This can be demonstrated by allowing the air inside the chamber to become moist and then rapidly expanding the moist air by either increasing the volume of the chamber or, as in our case, reducing the pressure. The temperature of the air decreases, and since the cooler air cannot hold as much moisture, condensation occurs. Water droplets form on the various nuclei present, e.g., dust particles, gas ions or molecules depending on the amount of expansion. If the expansion is great enough, water droplets form on the gas molecules and a cloud can be seen.

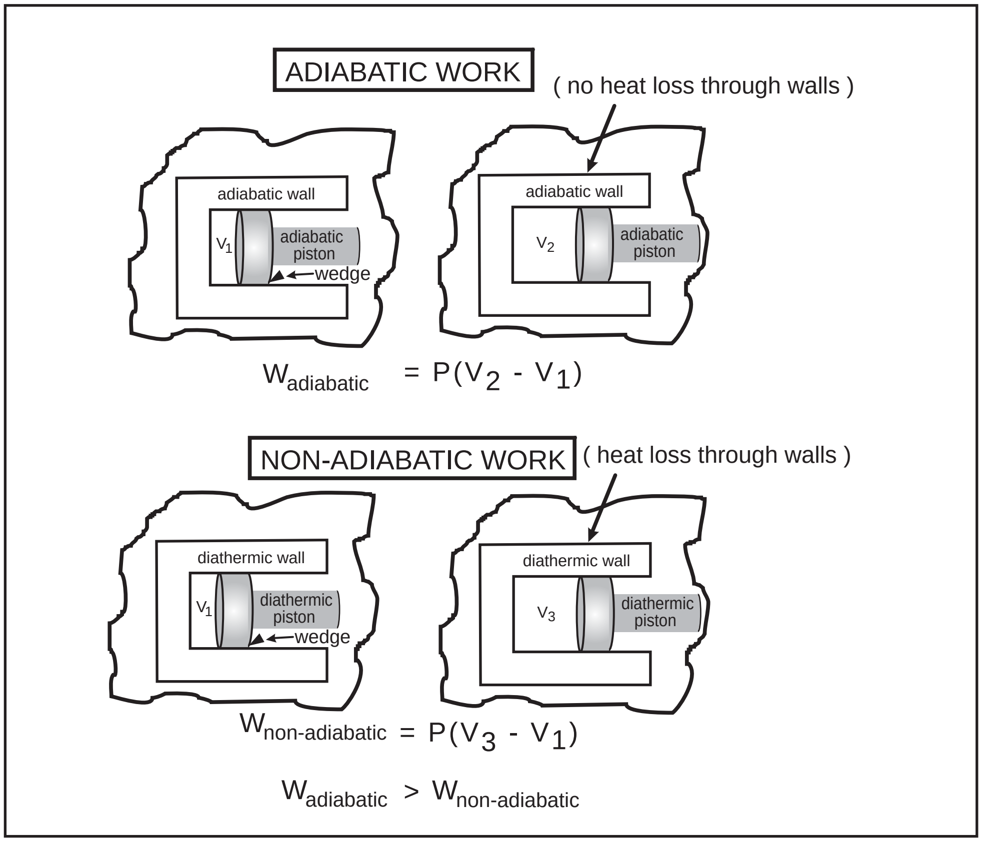


Figure 35. Classic illustrations of adiabatic and non-adiabatic work.

Demonstration #5: *Adiabatic Expansion of Moist Air*

The adiabatic expansion of a gas can be demonstrated in two ways with a simple vacuum system. As shown in Figure 36, the first way is to simply place a thermometer in a vacuum system and observe the temperature as the system is evacuated. If the pumping speed of the system is rapid, the temperature will drop by several degrees; however, it will begin to rise again as the system comes to thermal equilibrium. The second way to demonstrate adiabatic expansion is to produce a cloud chamber by introducing water vapor into a chamber and then pump it out. As the temperature of the air decreases, any water vapor present condenses, since cooler air cannot hold as much moisture. If the expansion is great enough water droplets form on the gas molecules, producing a cloud. The clouding effect can be made more dramatic by dimming the room lights and shining a light through the chamber during pumpdown.

Water can be introduced into the chamber by moistening the walls of the chamber before pumpdown or by introducing water vapor into an evacuated system. The latter can be achieved by placing a heavily moistened cloth over the up-to-air valve inlet for a few seconds when the system is partially evacuated and the pump turned off. Close the valve, turn on the pump, quickly dim the room lights and observe the cloud formation with a bright flash light.

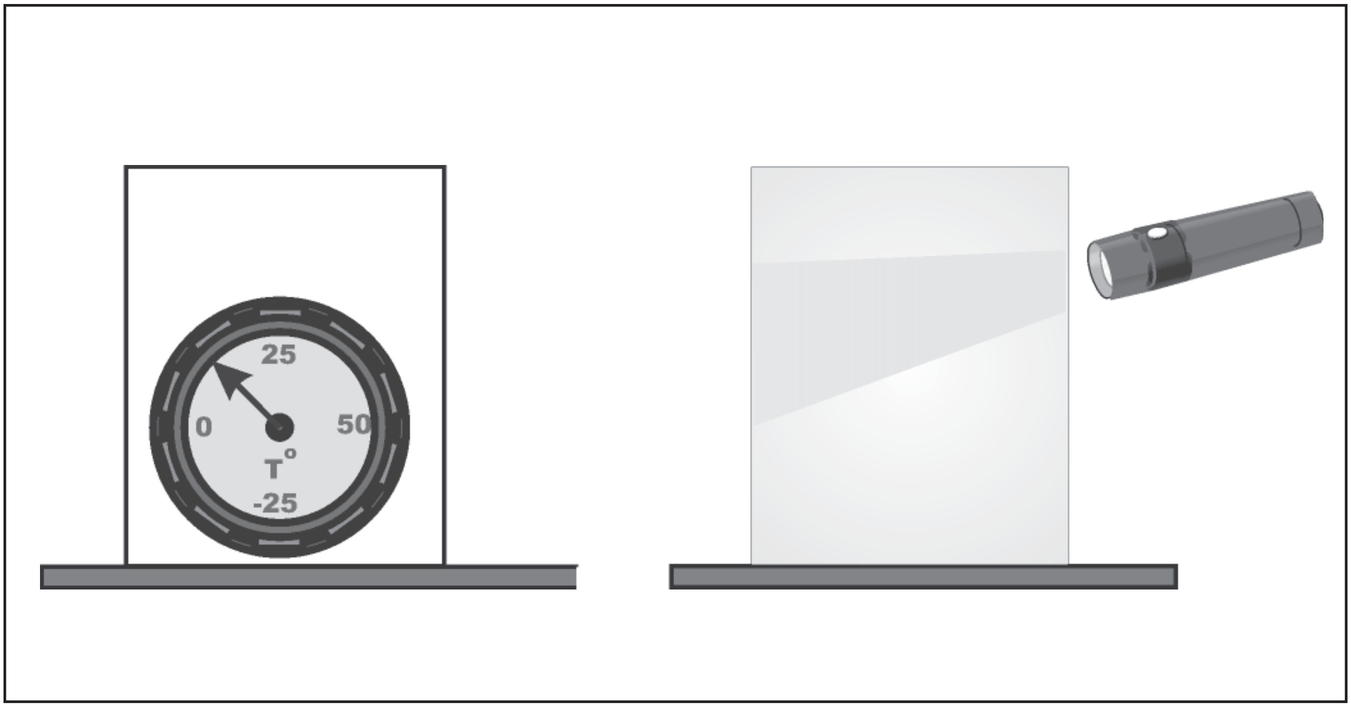


Figure 36. Demonstration of adiabatic expansion using (a) a thermometer and (b) a water vapor cloud.

IX. ENVIRONMENTAL ENGINEERING APPLICATIONS

A. Wastewater Treatment
B. Subsurface Remediation
C. Air pollution Control
D. Waste Minimization
E. Pollution Prevention

IX(A). Wastewater Treatment

Advanced Oxidation of Recalcitrant Organics in Wastewaters

The CAV-OX$_{TM}$ system is a commercial process for advanced oxidation of water with dissolved recalcitrant organic pollutants. The process, outlined below in Figure 37, is based on the premise that hydroxide free radicals can form from water molecules when cavitation occurs in a pump. These hydroxide free radicals begin a chain reaction whereby the recalcitrant organic compounds are broken down to smaller and smaller fragments. The smaller organics may be oxidized all the way to carbon monoxide and water or become small enough that they are readily biodegradable.

Complete destruction of the hazardous organic compounds is ensured by exposing the water to ultraviolet radiation immediately after the free radicals are formed in the cavitation chamber. The exposure to ultraviolet light ensures propagation of the radical chain reactions for a designated time for a particular contaminated water. A portion of the effluent stream leaving the UV light reactor is recycled (with some of the propagating free radicals) to the beginning of the process. The remaining portion is discharged as purified water.

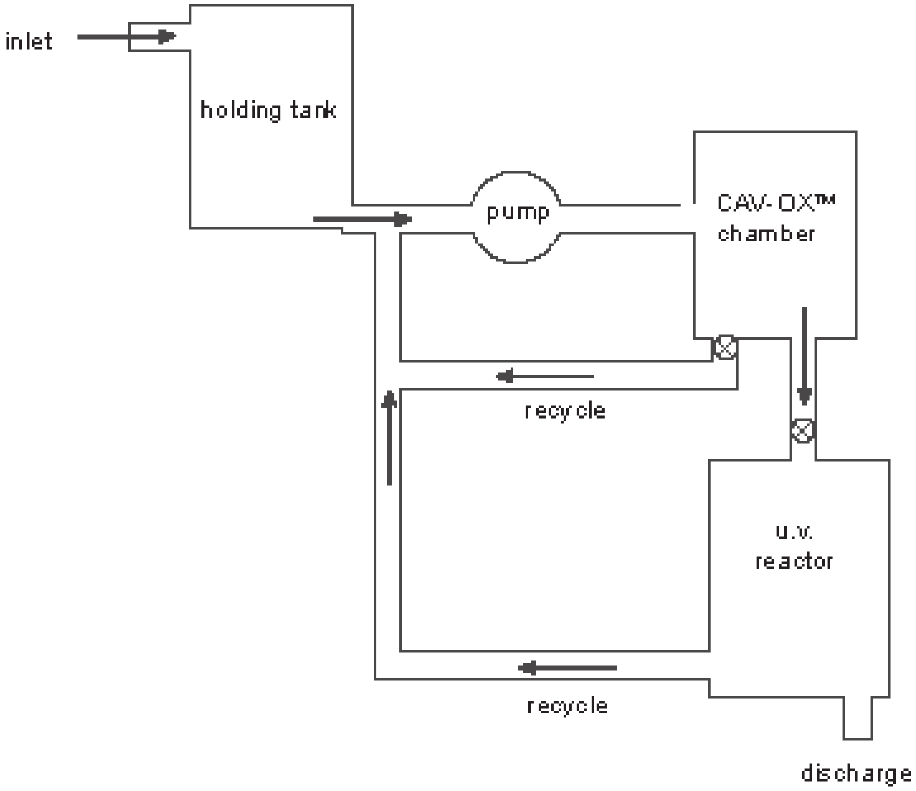
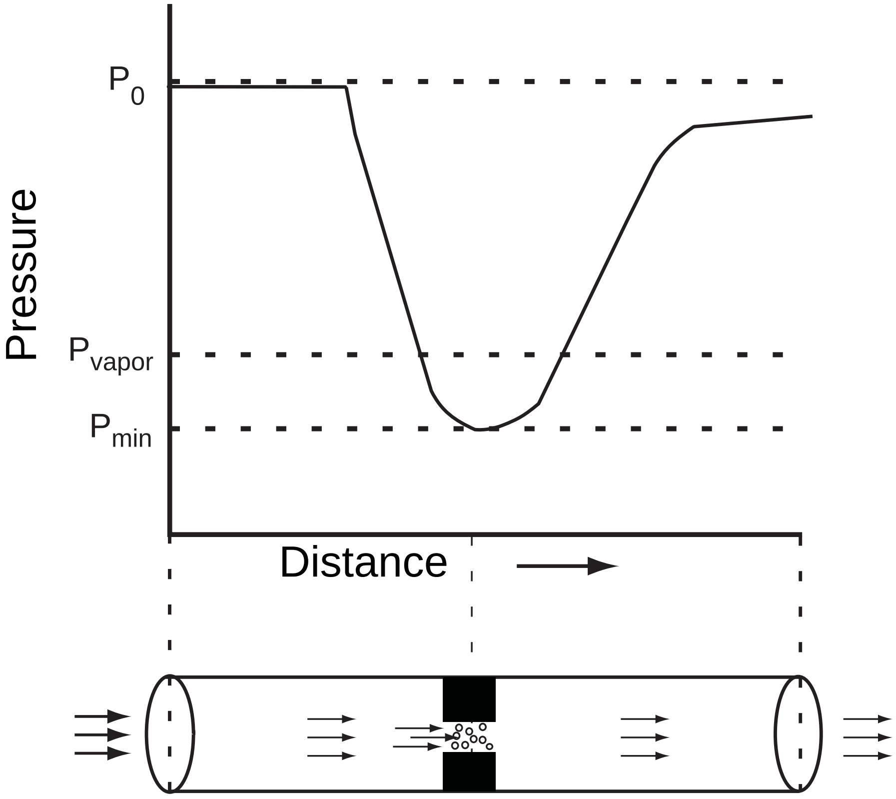


Figure 37. Schematic of CAV-OX™ advanced oxidation process for water treatment.

Cavitation is a phenomena that might occur when a liquid is forced to pass through a restricted (smaller) orifice, as shown in Figure 38. The velocity of the flowing liquid increases as it passes through the restriction. The increased velocity is accompanied by a decrease in pressure. If the pressure drops below the vapor pressure of the liquid, some of the liquid vaporizes forming cavities of gas (water vapor, in the present case) in the liquid.

As the fluid leaves the orifice, the velocity drops and the pressure rises. When the pressure rises above the vapor pressure of the liquid, the gas bubbles or cavities collapse back to the liquid state, releasing energy. The energy released is sufficient to excite the water molecules. These excited water molecules can break apart into hydrogen and hydroxide-free radicals. In an effort to appease their unpaired electrons, the high energy hydroxide-free radicals begin a series of chain reactions with the nearby dissolved organic molecules in the wastewater that are the target of the destruction technology.



1. The pressure of the liquid drops below its vapor pressure due to flow phenomena and forms cavities of gas in the moving liquid.
2. Some liquid vaporizes and forms cavities of gas in the moving liquid.
3. The gas cavities collapse back to the liquid state when the pressure rises above the vapor pressure beyond the constriction.

Figure 38. Schematic of the cavitation phenomena.

IX(B). Subsurface Remediation

Soil Vapor Extraction

Below the surface of the earth, there are several distinct geologic soil zones that can be defined by their water content. There is a dry zone right under the surface that normally has no water (except immediately after rainfall events) in the pore space between the sand and soil grains. Some distance below the surface a water table exists, which is the elevation below which all the pore spaces between the sand and soil particles are filled with water.

Immediately above the water table a zone exists where the soil void space is partially filled with water and partially filled with air. The water in this zone gets there via capillary rise action, pulling itself up the tiny spaces by means of surface tension in the small diameter pores. These subsurface zones are illustrated in Figure 39. Contamination in the subsurface may be found in any one or all of these three zones depending upon where the contaminant was released (such as a spill at the surface or a leaky underground storage tank). Different technologies must be used to remediate contamination in these different zones due to the different environment of the undesirable chemical. Soil Vapor Extraction is a technology that may be used to remove contamination from the vadose zone, which is defined as the subsurface zone that extends from the ground surface down to the water table. This technology can be used to remove organic contaminants that have low vapor pressures. Some organic compounds that may be removed by soil vapor extraction are listed in the Table 5.

A schematic of the technology is illustrated in Figure 40. It includes an extraction well to pump up contaminated groundwater. At the surface, it is connected to one of the types of vacuum pump listed in Table 6. The pump reduces the local pressure in the vadose zone to a value below the vapor pressure of the contaminant to be removed. This extraction well draws from the soil pore spaces and is surrounded by an array of several air injection wells. The injection wells pump air down into the vadose zone to provide an adequate supply of air to the soil pore spaces from which the extraction well draws its air. Once extracted from the subsurface, any liquid water suctioned up with the vapors is separated from the contaminated air. Ultimately, both the gaseous and liquid discharges must be sent to treatment or be destroyed. Final removal or destruction of the organic contaminant may be accomplished by subjecting the water and/or air streams to an above ground separation or destruction technology such as carbon adsorption or incineration.

Table 5. Some organic compounds removed by soil vapor extration

Volatiles	**Semi-volitiles**	**Hydrocarbons**
toluene	DCB (dichlorobenzene)	jet fuels
ethylbenzene	chlorobenzene	diesel fules
cyclobenzene	methanol	kerosene
PCE (tretrachloroethylene)	pyridine	gasoline
acetone	dimethylfuran	heavy naphthas
carbon tetrachloride		
TCA (triuchloroethane)		
benzene		
methylene chloride		
TCE (trichloroethylene)		

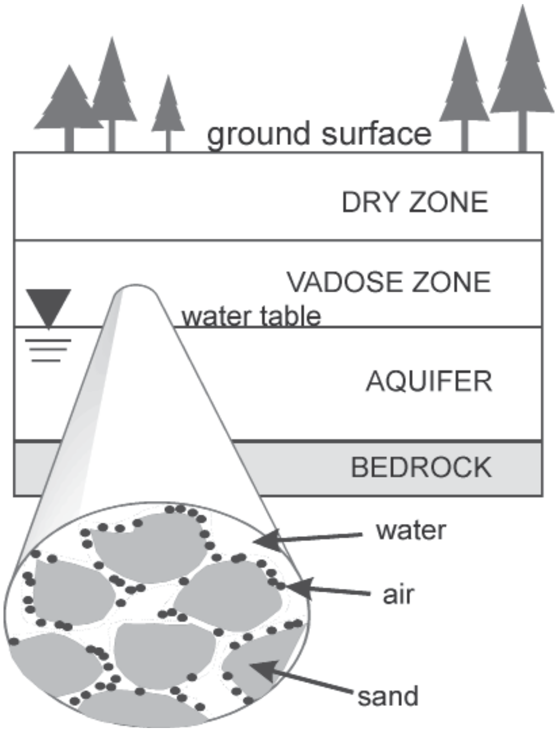


Figure 39. Subsurface profile of aquifer zones.

Table 6 Vacuum pumps for soil vapor extration

Liquid Ring Pumps

- •0-29 inches Hg (760 torr)
- •10-100 cubic feet per minute flow
- •used for remediation and permeability tests
- •achieves high vacuum without wearing out rotor by compressing water
- •uses mechanical seal plus O-ring gasket for zero leaks

Rotary Vane Pumps

- •used for air permeability tests of soil strata and low volume flow applications
- •27 inches Hg minimum pressure
- •no oil required when carbon vanes/blades are used

Regenerative Blower Pumps

- •low to moderate vacuums at high flow rates (large volume)
- •used for large jobs

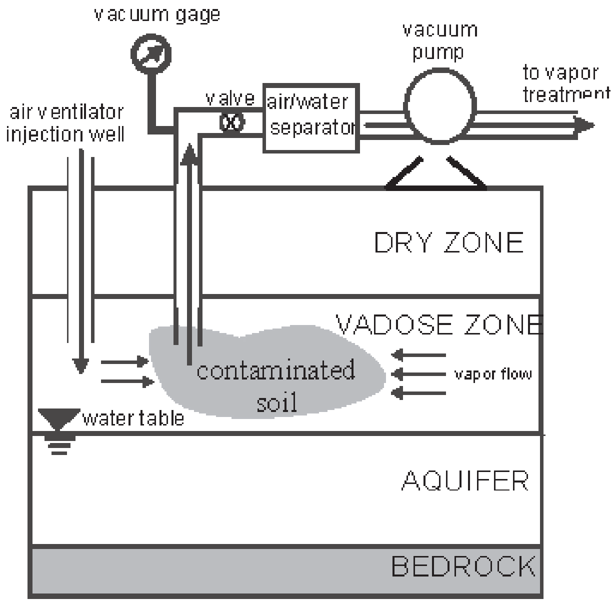


Figure 40. Schematic of a soil vapor extration process.

IX(C). Air pollution Control

Landfill Gas Recovery

The process of biological decay can be either aerobic (with oxygen) or anaerobic (without oxygen). In sanitary landfills, the decay process is primarily anaerobic. The municipal solid wastes disposed of in the landfill are covered and compacted each day, which keeps air (and, therefore, oxygen) away from the decaying wastes. The end products of anaerobic decay include the gases methane and carbon dioxide. Since gases take up more volume than either liquids or solids if they are allowed to accumulate in the covered landfill, they will build up pressure and may potentially cause the landfill to explode. To minimize the potential for such an explosion, it is necessary to remove the landfill gases. This is achieved by facilitating their migration through the buried garbage and soil layers and to vent them to the surface at the peripheral of the buried garbage. Because there are large amounts of methane gas which has an unpleasant odor, and because the combined carbon dioxide and methane mixture is combustible, it is desirable to collect the landfill gases and dispose of them or use them in a controlled manner.

The gas extraction process, as shown schematically shown in Figure 41, is similar to that used in soil vapor extraction, where venting wells are installed around the landfill. Each well is surrounded by several concentric layers of graded porous material (with increasing porosity) so that the gases will migrate toward the wells. The wells are tied together above the surface with a header conduit system that is ultimately connected to a vacuum pump. The pump reduces the pressure in the wells to further enhance gas migration and collection. The collected landfill gas is either torched on site or collected and sold to local power utilities to be used as a fuel.

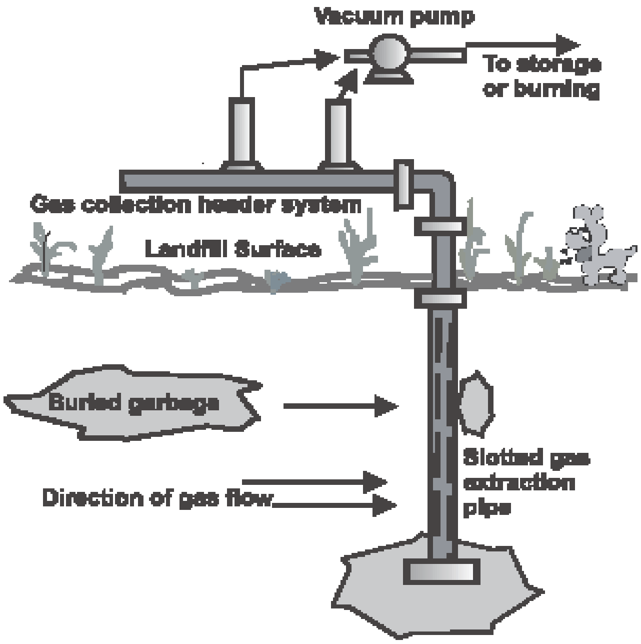


Figure 41. Schematic of landfill gas recovery well and collection system.

IX(D). Waste Minimization

Dewartering Filter for Sludges

Sludge may be organic, inorganic, or biological (better known today as "biosolids") in nature. Generically, they are mixed waste solids that result from various water, wastewater and industrial water treatment processes. Most sludges have solid contents that are 5% or less. This means that 95% or more is water. This high water content makes sludges unnecessarily expensive to dispose of because much of the disposal cost is related to transportation of the material. Transporting water mixed with the waste is just not economical nor good use of our water resources. The water can be separated, treated and recycled through a treatment facility. Therefore, to reduce the transportation costs of the sludges, most are dewatered at the plant before disposal.

There are several commercially available technologies currently used for dewatering various different types of sludges. These include centrifugation, floatation, compression settling and vacuum presses. One such device is a vacuum belt press as shown in Figure 42. Here, the wet sludge is spread out on a wide conveyor belt which passes over a suction plate under which a vacuum has been created. At the same time that the wet biosolids are suctioned from below, it is compressed by another belt or a roller from above. (This process uses the same principle as suction filtration processes often performed in chemistry labs with aspirators or vacuum pumps.) Using one or more of these dewatering technologies in series, the solids content of sludges may be increased up to 20% solids. The economics of dewatering in most cases are more favorable than those of disposing of the wet sludge.

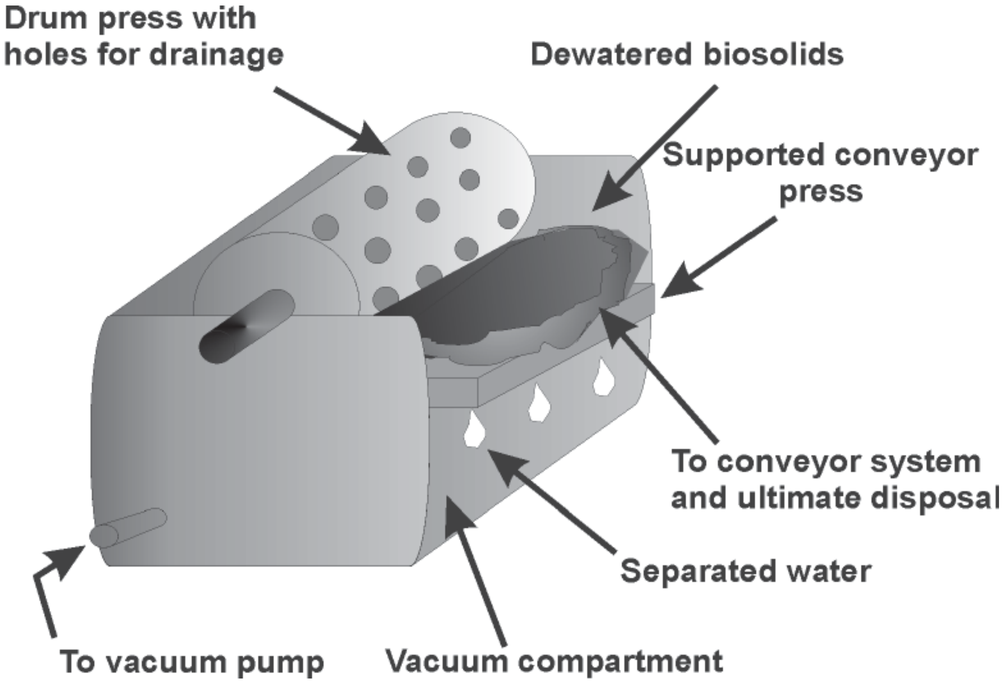


Figure 42. Schematic of vacuum dewatering system for biosolids.

IX(E). Pollution Prevention

Spray Painting with Supercritical Fluids

One application of a supercritical fluid is solvents for paints that are applied with sprayers. Supercritical carbon dioxide can be used as a carrier for the paint that is to be applied by a pressurized spray device.

Figure 43 shows a schematic of a basic supercritical fluid painting system. Pressurized liquid carbon dioxide (or any other supercritical fluid carrier) is pressurized with an appropriate pump. The CO_2 becomes supercritical in line. As the fluid is pumped, small amounts of a paint pigment are metered into the line. After a sufficient mixing length, the "paint" is dispensed through a fine high pressure nozzle designed for the application As shown in Figure 45, pressurized liquid carbon dioxide is brought above it critical temperature and pressure (1071 psi and 31^0C).

An organic solvent can be used as an alternate carrier fluid. As the spent organic solvent becomes contaminated with paint pigment it must be collected for disposal, recycling, or reuse. Most organic solvents are somewhat harmful to human health. If it is a volatile liquid, it offers at least two exposure routes to humans: inhalation and adsorption through the skin during paint application. The use of supercritical carbon dioxide as the pigment carrier fluid eliminates exposure to hazardous chemicals used as solvents in this application.

The environmental advantage is that carbon dioxide returns to its vapor state when the paint/supercritical carbon dioxide mixture is vented through a nozzle as the paint is applied to the object desired. The vented carbon dioxide gas is generally considered to be nontoxic for humans and the environment, but may impact global warming.

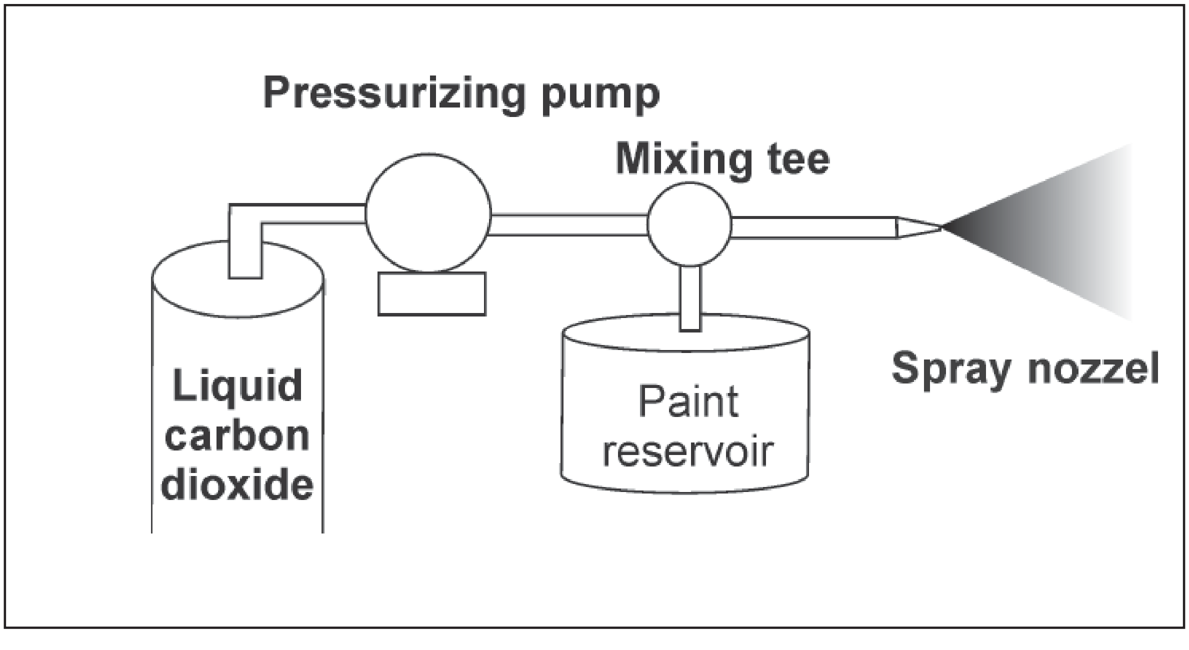


Figure 43. Schematic of a supercritical carbon dioxide painting application system.

X. APPENDIX

A. Mean Free Path

B. Vacuum Pumps
   1) Vacuum Safety
   2) Mechanical Vacuum Pump Operation
   3) Mechanical Vacuum Pump Maintenance and System Considerations

C. Total Pressure Gauges

D. Additional Demonstrations / Experiments

E. Additional References

F. Units and Vacuum Formulas

G. Conversion Factors

H. Units of Pressure

I. Drawings for Hand Held Vacuum Plates

Appendix A - Mean Free Path

The distance a molecule travels before it suffers a collision with another molecule.

$$\lambda = \frac{1}{2^{\frac{1}{2}} \pi\, d_0^2 n}$$

where: d_0 = molecular diameter and n = gas density

For air at room temperature

$$\lambda(mm) = \frac{6.6}{P}$$

where P is the pressure in Pascals

Low Pressure Properties of air

{ John E. O'Hanlon, *A User's Guide to Vacuum Technology*, p. 11 (John Wiley & Sons, New York, 1980) }

Pressure (Pa)	η (meters^{-3})	δ (meters)	λ (meters)
1.01x10^{5} (760 Torr)	2.48x10^{25}	3.43x10^{-9}	6.5x10^{-8}
100 (0.75 Torr)	2.45x10^{22}	3.44x10^{-8}	6.5x10^{-5}
1(7.5mTorr)	2.45x10^{20}	1.60x10^{-7}	6.5x10^{-3}
10^{-3} (7.5x10^{-6} Torr)	2.45x10^{17}	1.60x10^{-6}	6.64
10^{-5} (7.5x10^{-8} Torr)	2.45x10^{15}	7.41x10^{-6}	664
10^{-7} (7.5x10^{-10} Torr)	2.45x10^{13}	3.44x10^{-5}	6.6x10^{4}

where η is the particle density, δ is the average molecular spacing, and λ is the mean free path.

Appendix B - Vacuum Pumps

Vacuum Pumps fall roughly into one of three categories: positive displacement pumps, momentum transfer or exchange pumps, and capture pumps. Figure 46 shows the pressure operating ranges for the most commonly used vacuum pumps.

A positive displacement pump, shown in Figure 47, is a mechanical mechanism that moves a volume of gas from the pump's inlet port to its outlet port. Positive displacement pumps can either be oil sealed or oil free.

The two commonly used momentum transfer pumps shown in Figure 48 are the diffusion pump and turbo molecular pump. During operation, a component of these pumps collides with the gas molecules , forcing the molecules to move to their exhaust ports. In the case of the diffusion pump, the colliding part is a vapor stream of oil and in the case of a turbomolecular pump the colliding part is a turbine blade.

The distinguishing feature of capture pumps is that they do not exhaust the gases which they pump, but retain the gases either permanently or temporarily. Also, these pumps are selective in the type of gas they pump. Examples of this type pump are shown in Figure 49. They include: titanium sublimation pumps, sputter-ion pumps, cryosorption pumps, and cryogenic pumps.

A more detailed discussion of vacuum pumps can be found in the AVS Education Committee book series, Vacuum Technology: A Beginning , by Harland G. Tompkins.

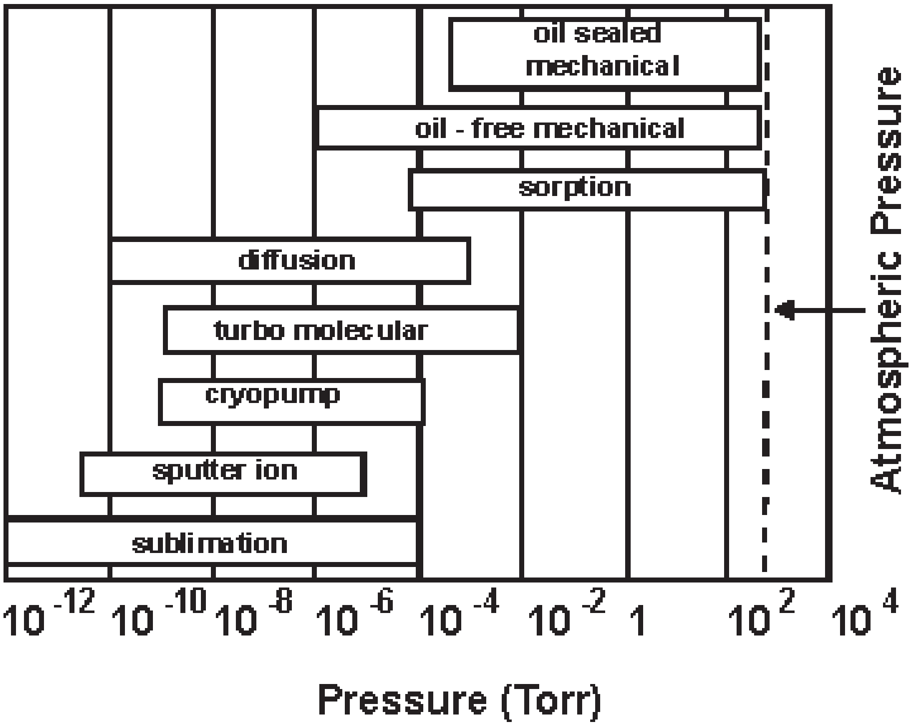


Figure 44. Approximate operational ranges for various vacuum pumps.

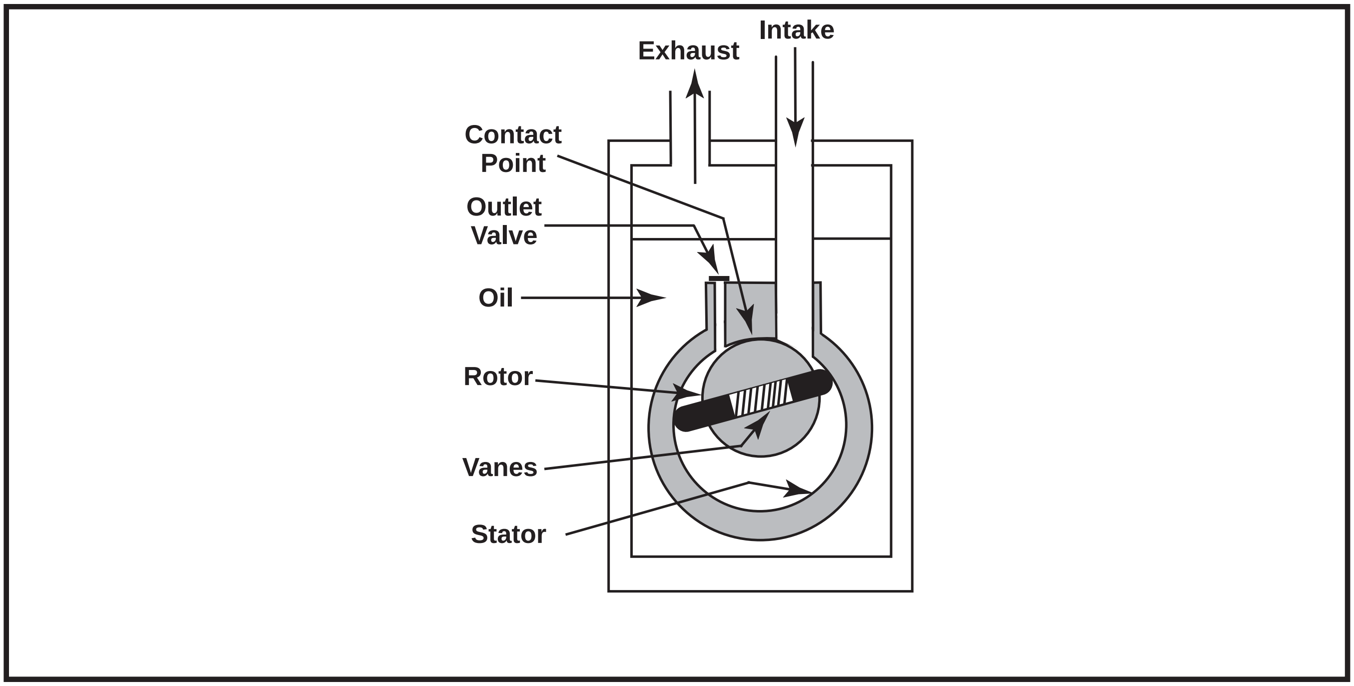


Figure 45 Positive displacement pump - oil-sealed mechanical pump.

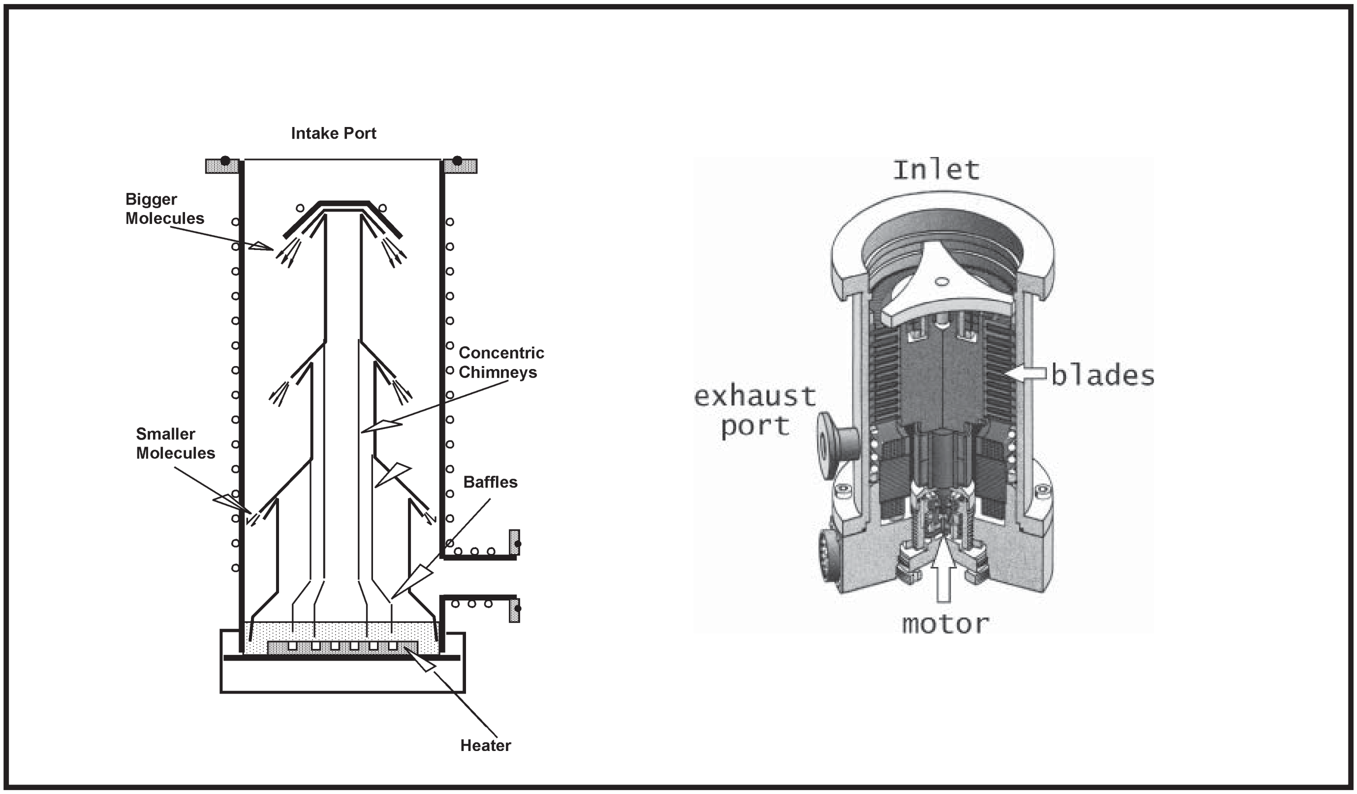
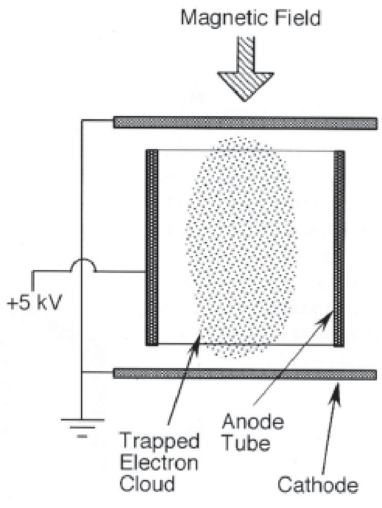
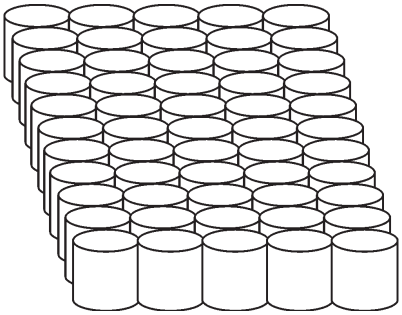


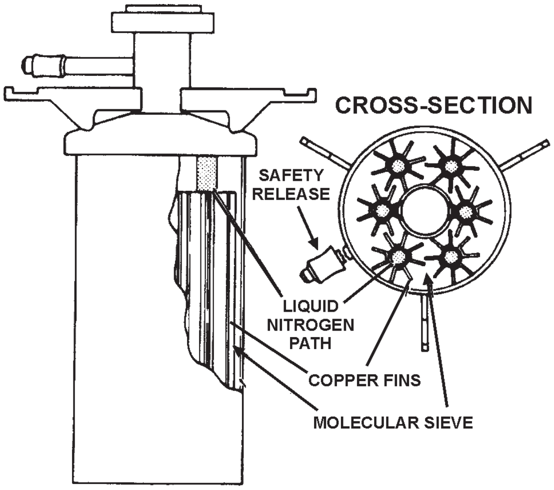
Figure 46. Momentum Transfer Pumps



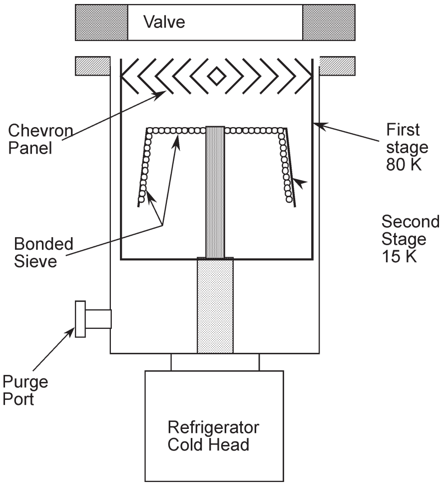
Sputter Ion Pump
(single element)



Sputter Ion Pump
(multiple elements)



Cryosorption Pump



Cryogenic Pump

Titanium Sublimation Pump

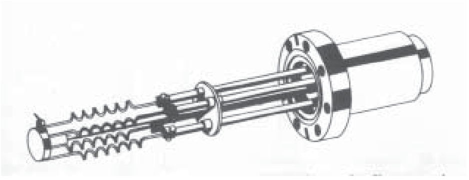


Figure 47. Capture Pumps

Appendix B(1)- Pump and Vacuum Precautions

A. OIL EXHAUST

Oil exhaust from mechanical vacuum pumps can result in three possible hazardous conditions. One, oil can drip and accumulate causing slippery areas as well as fire hazards. Secondly, the exhaust which lingers in the air will be either hydrocarbons, silicon compounds, fluorocarbons. If such exhaust is breathed over an extended period of time it can cause serious respiratory difficulties and ailments. For some people the time period can be relatively short. Third, fumes from processes in the main work chamber can end up in the mechanical pump exhaust. Depending on the process, these fumes can be lethal.

B. DRIVE BELT

A major hazard common to all machines that are operated by an electric motor and driven by a belt is associated with the exposed drive belt. Clothing, rags, paper, and any lose object can get caught in the drive belt causing injury to personnel and to the pump itself. Also in dry areas static electricity can build up and pull loose objects into the drive mechanism from considerable distances.

C. IMPLOSIONS

Vacuum systems, especially large surface area systems, must support enormous forces due to atmospheric pressure. Weak sections of a vacuum wall can implode, rupturing in the process, sending flying debris in all directions. The following precautions should be taken:

1. Avoid local stress on chamber by supporting heavy side arms and eliminating local heating and hot spots.

2. Use implosion shields around the system.

3. Wear safety glasses.

4. Avoid using hand tools and long handled implements around evacuated glass and plastic chambers.

5. Follow chamber manufacturer’s instructions regarding the use of solvents and vacuum greases.

Appendix B(2) - Mechanical Vacuum Pump Operation

The components and operation of a positive displacement oil sealed mechanical vacuum pump are shown in Figures 48 and 49, respectively In the basic operation the rotor rotates in a non-concentric pattern in such a way that the spring loaded sliding vanes trap and hold a volume of gas from the intake port and transports it to the exhaust port. As the volume moves towards the exhaust port it becomes compressed and if the resulting pressure is sufficient (~ 850 - 1000 Torr) the outlet valve will open and allow the trapped volume of gas to escape from the pump. The role of the oil in a oil sealed pump is threefold: lubrication, sealing, and cooling.

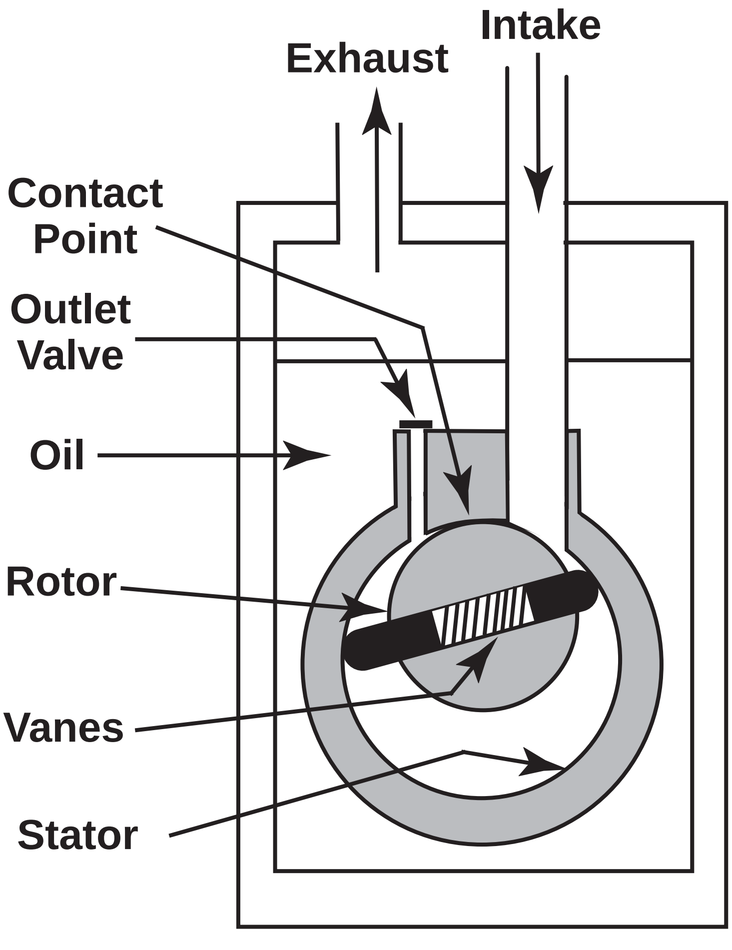


Figure 48. Schematic of a rotary vane oil-sealed mechanical vacuum pump.

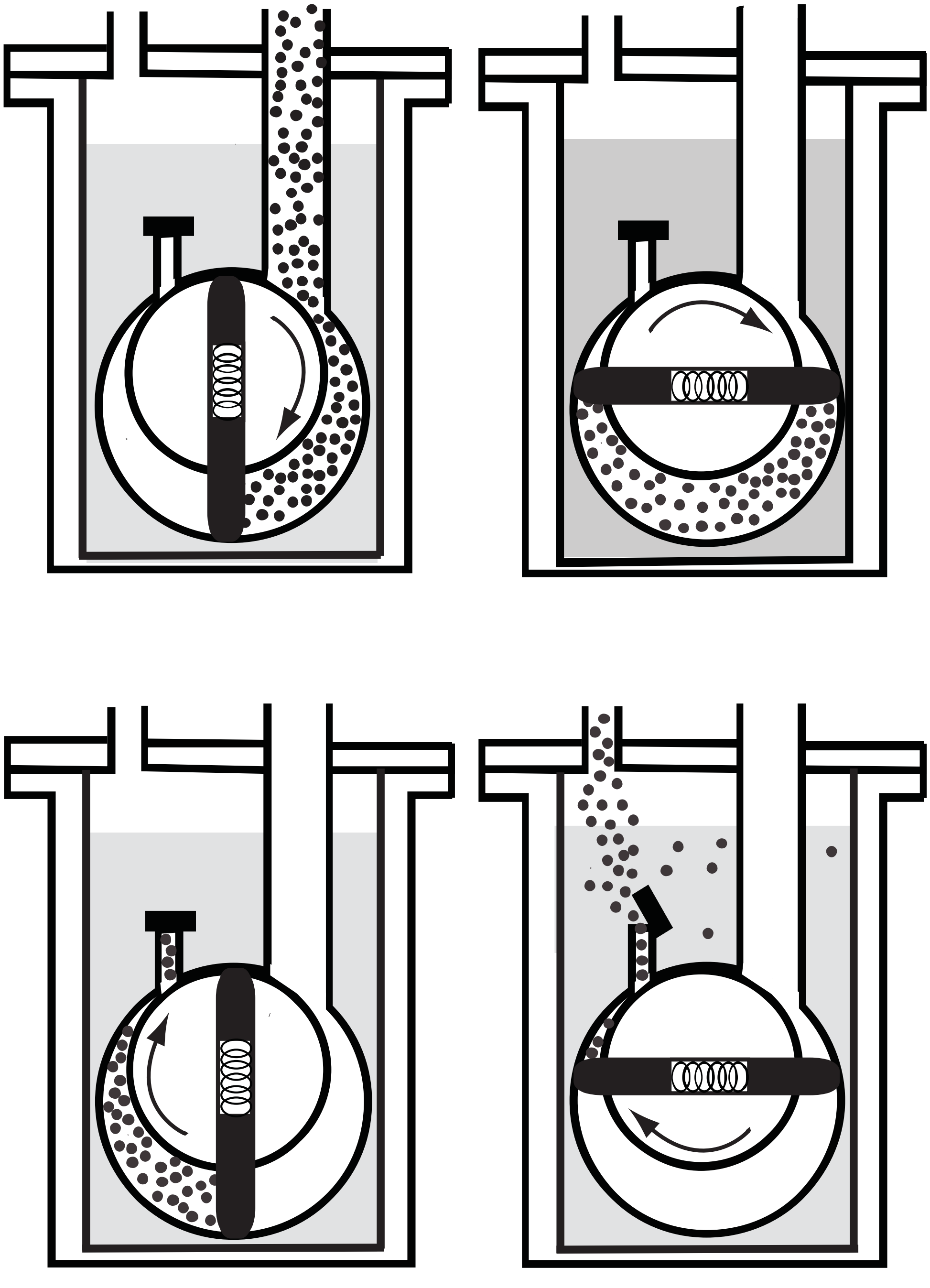


Figure 49. The principle of operation of a rotary vane oil-sealed mechanical vacuum pump.

Appendix B(3) - Mechanical Vacuum Pump Maintenance and System Considerations

There are a few useful tips that are important when using a mechanical pump. These will help in making the experiments easier to run and to prevent problems in the pumpdown and up-to-air cycles. Some, but not all, of this information can be found among the literature that comes with pumps and vacuum jars.

(1) Put oil into your pump. Nearly all mechanical pumps have a sight glass to show the oil level like the one in Figure 50. For the proper level, check the manufacturer's instructions.

(2) When constructing your vacuum station, avoid harsh bends and constrictions. These have the effect of lowering the pump's efficiency and pump-down times will be longer and the final pressure of the chamber will be higher. Use the shortest length of vacuum hose possible between the vacuum jar and the valve leading from the pump. Use hose-clamps when the vacuum hose terminates onto a barbed or tapered connector.

(3) Our experience has indicated that for these systems, pump-down time to the base pressure (ca. 30" Hg) is on the order of one minute. If this time is exceeded significantly or the equipment is making strange or labored sounds, vent the pump, shut it off and check your system for leaks. If none are found be sure to check that the pump is operating according to the criteria given by the manufacturer.

(4) After extended use, it is normal for pump to emit a smell due to the pressure of hot oil.

(5) If your pump is belt driven, after extended use check the belt tension For proper tension and adjustment, check the manufacturer's instruction that came with the pump.

(6) After experiments which involve the pumping of water vapor, check the color of your oil - it may appear cloudy due to the mixing of water with the oil. If the cloudiness does not begin to/or completely disappear after operating the pump for an extended length of time you may need to change the oil. As a matter of fact, any time the oil (as viewed through the glass window on the front of the pump) shows a significant color change which persists through a number of pumping cycles it is best to change it.

Figure 50. Oil level in vacuum pump sight glass.

Appendix C - Total Pressure Gauges

In order to know the pressure in a vacuum system some kind of pressure sensing device or gauge is required. There are many types of vacuum gauges, but, as shown in Figure 53, no single gauge can accurately measure ranging from atmosphere to below 10^{-11} Pa (10^{-13} Torr).

Vacuum gauges fall into two categories: those that measure pressure by calculating the force exerted on a surface by the environment (direct reading) and those that measure pressure dependent properties of gases (indirect reading). Direct reading gauges usually rely on a mechanical mechanism's response to pressure. An example of this, shown in Figure 54, is a diaphragm gauge which contains a pressure-sensitive surface that undergoes a change in shape or position when evacuated. This change in turn causes a mechanical pointer to be deflected. Other common types of direct reading gauges are the Bourdon tube and the capacitance manometer. The latter is just a diaphragm gauge in which the deflection of the diaphragm changes the capacitance between it and a fixed counter electrode. Direct reading gauges are generally used to measure pressures from atmosphere down to 10^{-3} to 10^{-5} Torr

Two commonly used indirect reading devices are the thermal conductivity and ionization gauges, shown schematically in Figure 55. Examples of thermal conductivity gauges are the thermocouple and Pirani gauges. These gauges measure the heat transfer between two surfaces at different temperatures. Heat transfer is pressure dependent. An ionization gauge uses electron impact to ionize surrounding gases. The electrons are generated with a hot filament. The positive ions thus created produce a current in an ion collector that is proportional to the surrounding pressure.

A more detailed discussion of vacuum pumps can be found in the AVS Education Committee book series, Vacuum Technology: A Beginning , by Harland G. Tompkins.

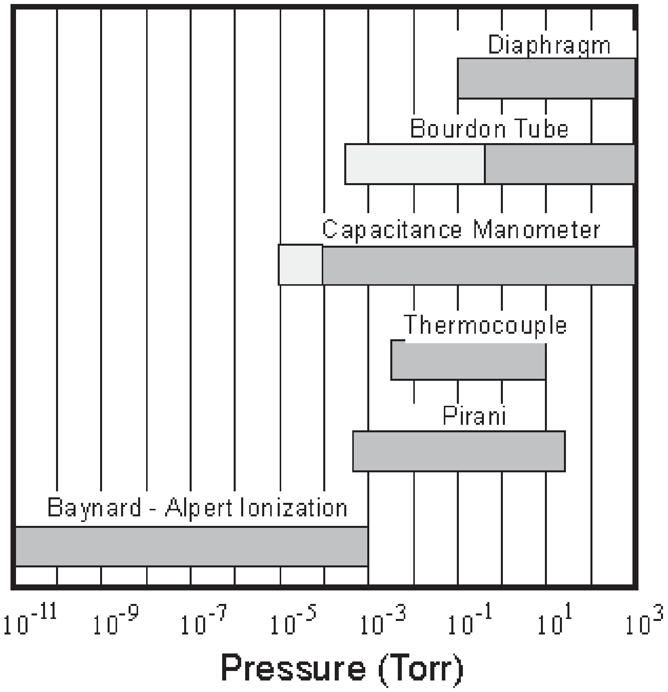
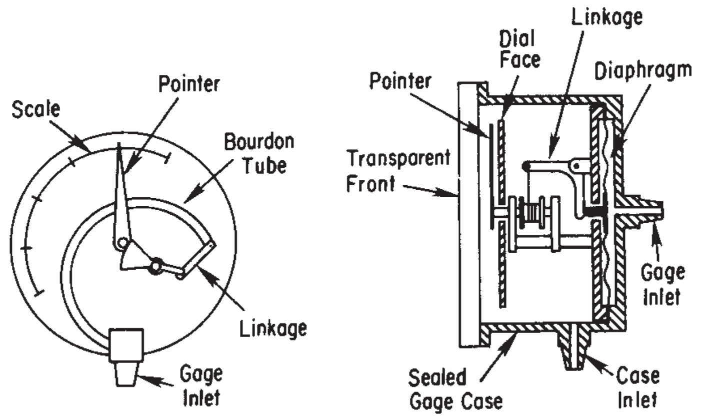


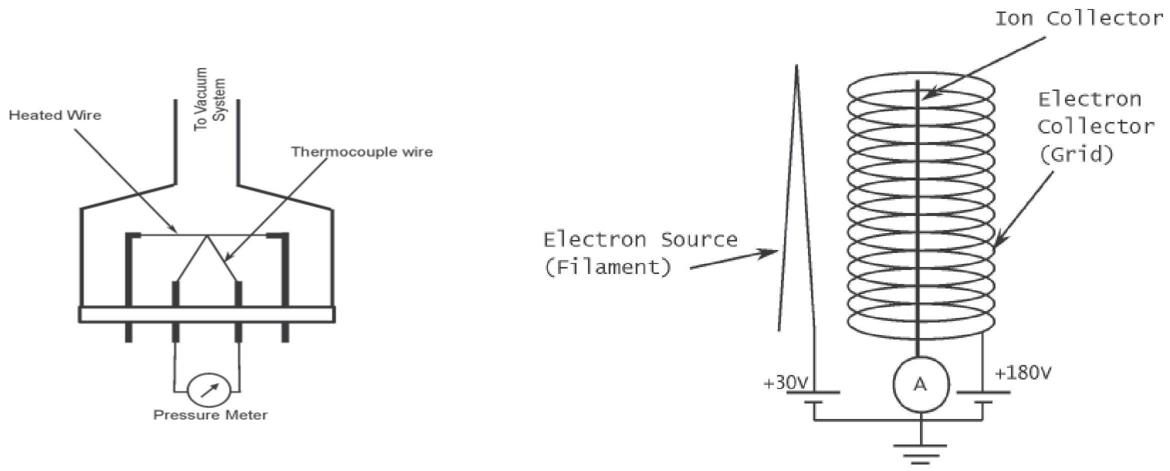
Figure 51. Pressure ranges for total pressure gauges.



Bourdon Gauge

Diaphragm Gauge

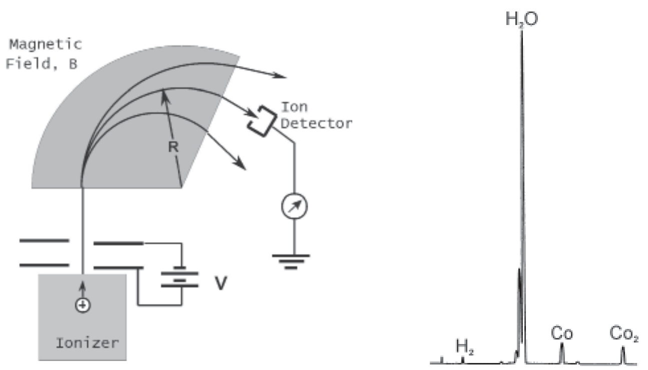
Figure 52. Direct reading gauges.



Thermocouple Gauge

Ionization Gauge

Partial Pressure Analyzer
(mass spectrometer)



mass spectrum
of gases in
a vacuum
chamber

Figure 53. Indirect reading gauges.

Appendix D- Additional Demonstrations/Experiments

1. **The sublimation of ice at various pressure.**

Vacuum systems are used by the food industry for freeze drying. Moisture is removed by sublimation. Use an ice cube to demonstrate sublimation. As the pressure drops, whiskers grow out from the ice and visible clouds of fog can be observed. Weighing the ice before and after expose to reduced pressure shows the weight loss due to sublimation.

2. **Thin film evaporation.**

Thin metallic films can easily be grown using a low melting point metal such as indium. A chamber with two electrodes is required. The metal can be evaporated from a tungsten wire or boat. Be sure to use proper eye protective gear when looking at a bright filament. Also, shield the walls of the vacuum chamber from the depositing metal, Issues to investigate include the effect of substrate surface preparation on adhesion, the effect of evaporation rates on the physical properties of deposited films, and the determination of film thickness.

3. **Buoyancy of air - Archimedes' Principle.**

Construct a simple balance in a vacuum chamber with a glass bulb on one end and counter weights on the other end of the beam. Under vacuum the system becomes unbalanced because air no longer buoys up the bulb. The true weight of an object can only be determined in a vacuum. Determine the error involved in weighting the bulb in air.

4. **Others.**

A number of advanced experiments, as well as those covered in this workbook, can be found at the AVS Education Committee internet link http://www.avs.org.

Appendix E - Additional References

Experimental Vacuum Science and Technology, American Vacuum Society Education Committee eds., Marcel Dekker, Inc., New York, 1973.

700 Science Experiments for Everyone, Compiled by UNESCO,Doubleday & Co., Inc., Garden City, New York.

A Users Guide to Vacuum Technology, J. O'Hanlon, Wiley, New York, 1989.

"Pumping Speed and Boyle's Law: Two Vacuum Experiments" , B.R.F. Kendall, *The Physics Teacher, Volume 34*, December 1996, pp 538-542.

"Educational Outreach at the 42nd National Symposium of the American Vacuum Society", *AVS Monograph Series M16*, Patricia A. Thiel and David E. Fowler eds., 1996.

"Pumps Used in Vacuum Technology", *AVS Monograph Series M9*, Harland G. Tompkins and Timothy A. Gessert, 2001.

"Vacuum Gauging and Control", *AVS Monograph Series M12*, Harland G. Tompkins, 1994.

Internet Resourses

Teacher Professional Development - http://learner.org

Modeling Software for Students -http://www.ncrel.org/engauge/resource/berkeley/modelit.htm/index.htm

States of Matter - http://chem4kids.com/files/matter_states.html

A Short History of Vacuum Terminology and Technology - http://www.mcallister.com/vacuum.html

Vacuum Experiments and Demonstrations - http://www.avs.org/education.workshopaspx and http://www.belljar.net

Appendix F - Units and Vacuum Formulas

Units of Pump Speed (Vol/Time)

1 Liter Sec^{-1} = 2.12 CFM
1 CFM = .47 Liter Sec^{-1}
1 Liter Min^{-1} = .035 CFM

Units of Gas Mass (PV)

1 Molar Volume = 22.41 Liters
(at standard conditions- STP)
1 Mole = $6.023\text{x}10^{23}$ Mols
1 Liter-Atmos = $2.68\text{x}10^{22}$ Mols
1 Std. cc = $2.68\text{x}10^{19}$ Mols
1 Torr-Liter = $3.52\text{x}10^{19}$ Mols
1Std. cc = .76 Torr-Liter
1 Std. cc = 1 Atmos cm^3
1 Cubic Foot = $7.6\text{x}10^{23}$ Mols
(Standard Conditions are 1 Atmosphere at 273K)

Units of Throughput (Q=PV/Time)

(PV = Work; Work/Time = Power)
1 Std. cc Sec^{-1} = 760 Micron-Liters Sec^{-1}
= 1.6 Torr CFM
1 Torr CFM = .62 Std. cc Sec^{-1}
= 472 Micron-Liters Sec^{-1}
1 Micron-Liters Sec^{-1} = $1.32\text{x}10^{-3}$ Std. cc Sec^{-1}
= $2.12\text{x}10^{-3}$ Torr CFM

Vacuum Formulas

A. Conductance = Volume of Gas Flowing Through Orifices and Pipes Per Unit Time

1. Molecular Flow (below 10^{-3} Torr) through orifices

   C_0 = 11.6 Area in liters per second

   $$11.6\,\frac{\pi D^2}{4}$$ D in cm

2. Molecular Flow (below 10^{-3} Torr) through pipes

   $$C_p = 11.6\,\frac{D^3}{L}$$ in liters per second

   D and L in cm

3. Viscous Flow (10^{-1} to 760 Torr) through pipes

$$C_v = 180\frac{D^4}{L}\bar{P}$$

C_v in liters per second

D in cm
L in cm
Average $\bar{P}$ in Torr

4. Conductances in Series = C_{Total}

$$C_{TP} = C_1 + C_2 + C_n.....$$

5. Conductances in Parallel = C_{Total}

$$\frac{1}{C_{TP}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_n} + ...$$

6. Resultant Pumping Speed at a Vacuum System

$$\frac{1}{R} = \frac{1}{S} + \frac{1}{C}$$

C = Conductance

R = Liters per Second
S = Pump Speed (in liters per sec)

B. Fundamental Equation for a System in Equilibrium

Q = PS
Q = System gas load in Torr-liters sec^{-1}
P = Equilibrium pressure in Torr
S = Effective (resultant) pumping speed in liter sec^{-1}

(Q) is also called throughput and is work per unit time (PV/t).

C. Ideal Gas Equation

PV = nRT

P = Torr
V = Liters
n = No. of Moles
R = Molar Gas Constant
T = Kelvin

or:

PV = nkT

P = Torr
V = Liters
n = No. of Molecules
k = Boltzmann's constant
T = Kelvin

D. Mean Free Path

$$L = \frac{5x10^{-3}}{P_{Torr}}$$ (in centimeters)

E. Molecular Velocity

a) $$\overline{V} = 1.5\text{x}10^{-4}\sqrt{\frac{T}{M}}\ \text{cm}\cdot\text{sec}^{-1}$$

T = K
M = Molecular Weight

b) $$\overline{V} = \sqrt{\frac{8kT}{\pi M}}\ \text{cm}\cdot\text{sec}^{-1}$$

k = Boltzmann's Constant
T = K
m = Molecular Mass in Grams

F. Pumpdown (Roughing) Formulas

a) $$P = P_o\, e^{\frac{-St}{V}}$$

P_0 in Torr (original pressure)
P in Torr
S in liters per second
V in liters
t in seconds

b) $$t = \frac{V}{S} \ln \frac{P_o}{P}$$

Appendix G - Conversion Factors

CONVERSION FACTORS: Conversion factors preceded by a dot (•) are exact and serve to define one unit in terms of another. For example, the factor 0.3048 m/ft defines the foot as exactly 0.3048 m.

---

1. Length

• 10^2 cm / m
• 10^3 m / km
• 2.54 cm / in
• 12 in / ft
• 5,280 ft / mile
• 0.3048 m / ft
• 1.609344×10^3 m / mile
• 1.609344 km / mile
1.49598×10^{11} m / A.U.
9.461×10^{15} m / light-year
3.084×10^{16} m / parsec
• 10^{-6} m / µm (or m/micron)
• 10^{-10} m / Å
• 10^{-15} m / fm

2. Volume

• 10^{-3} m^3 / liter
• 10^3 cm^3 / liter
0.94635 liter / quart
3.7854×10^{-3} m^3 / gallon

3. Time

(a day is a mean solar day; a year is a sidereal year.)
• 3,600 sec / hr
• 8.64×10^4 sec / day
365.26 day / year
3.1558×10^7 sec / year

4. Speed

• 0.3048 (m / sec) / (ft / sec)
1.609×10^3 (m / sec) / (mile / sec)
0.4470 (m / sec) / (mile / hr)
1.609 (km / hr) /(mile / hr)

5. **Acceleration**
   - 0.3048 (m / sec^2) / (ft / sec^2)

6. **Angle**
   - 60 second of arc(“) / minute of arc(‘)
   - 60 minute of arc(‘) / deg
   - 180/π (≅ 57.30) deg / radian
   - 2π (≅ 6.283) radian / revolution

7. **Mass**
   - 1,000 gm / kg
     453.59 gm / lb
     0.45359 kg / lb
     2.2046 lb / kg
     1.66053x10^{-27} kg / amu
     6.0222x10^{26} amu / kg
     6.0222x10^{23} amu / gm

8. **Density**
   - 103 (kg / m^3) / (gm / cm^3)
     16.018 (kg / m^3) / (lb / ft^3))
     1.601810^{-2} (gm / cm^3) / (lb / ft^3)

9. **Force**
   - 10^5 dyne / N
   - 10^{-5} N / dyne
     4.4482 N / lbf
     (1 lbf = weight of 1 pound at standard gravity [g = 9.80665 m / sec^2])

10. **Power**
    - 746 W/horsepower

11. **Temperature**
    - 1.00 °F / °R

    - 1.00 °C / K
    - 1.80 °F / °C
    - 1.80 °R / K
    - T(K) = T(°C)+273.15
    - T(°C) = [T(°F)-32]
    - T(K) = T(°R) / 1.80

12.Energy

- 10^7 erg / J
- 10^{-7} J / erg
- 4.184 J / cal
- 4,184 J / kcal
- 10^3 cal / kcal

(The kilocalorie [kcal] is also known as the food calorie, the large calorie, or the Calorie.)

1.60219×10^{-19} J / eV

1.60219×10^{-13} J / MeV

- 10^6 eV / MeV
- 3.60×10^6 J / kW hr

4.20×10^{12} J / kiloton

4.20×10^{15} J / megaton

0.04336 (eV/molecule)/(kcal/mole)

23.06 (kcal/mole) / (eV/molecule)

(eV is the symbol for energy unit know as an electron volt)

13. Electrical quantities

(Note that 2.9979 is well approximated by 3.00.)

Charge: 2.997910^9 esu / C

Current: 2.9979×10^9 (esu / sec) / A

Potential: 299.79 V / statvolt

Electric field: 2.9979×10^4 (V / m) / (statvolt / cm)

- Magnetic field: 10^4 G / T
- Magnetic flux: 10^8 G cm^2 / Wb
- Pole strength: 10 cgs unit / michel [cgs unit = $(erg\ cm)^{-2}$; michel=AM]

Appendix H - Units of Pressure

	Pascal (Pa) (N/m^2)	1 Torr (1 mm Hg)	Std Atm (atm)	Millibar (mbar)	Dyn/cm^2	lb/in^2
1 Newton/square meter (N/m^2)	1	$7.5x10^{-3}$	$9.87x10^{-6}$	10^{-2}	10	$1.45x10^{-4}$
1 Torr (1 mm Hg)	133	1	$1.32x10^{-3}$	1.33	1,330	$1.93x10^{-2}$
1 standard atmosphere (atm)	101,000	760	1	1,010	1,101,000	14.7
1 millibar (mbar)	100	0.75	$9.87x10^{-4}$	1	1.000	$1.45x10^{-2}$
1 dyne/square cm (dyn/cm^2)	10^{-1}	$7.5x10^{-4}$	$9.87x10^{-7}$	10^{-3}	1	$5.15x10^{-5}$
1 pound / square inch (lb/in^2)	$6.90x10^3$	51.7	$6.80x10^{-2}$	69	$6.90x10^4$	1

Appendix I - Drawings for hand Held Vacuum Plates