

## The Gas Laws: Simple and Inexpensive Demonstrations

Northern California Chapter –  
American Vacuum Society

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## Who are we?

- The Education Committee of the Northern California Chapter of the AVS (American Vacuum Society)
- We are S.F. Bay area engineers, scientists, and technologists – not professional teachers.
- We have numerous active programs supporting science education both locally and nationally.
- Please stop by our table in the exhibition area (Table G) for more information about our programs (and some free materials).

## Why the Gas Laws?

- The scientific investigation and understanding of the properties of the third state of matter – gas was a critical factor enabling the industrial revolution.
- The first “steam engine” was actually a vacuum based machine!
- The Gas Laws (Boyle’s, Charles’s, Ideal) have more practical importance than the well known  $E=mc^2$
- Critical for most of today’s technical education and jobs.

## Objectives

- The objective of this presentation is to provide ideas for demonstrating the fundamental laws determining how gases behave.
- These demonstrations use only low cost, readily available equipment but are capable of showing both **qualitative and quantitative** gas behavior.
- These demonstrations are student safe, allowing “hands on” learning.
- **Establish the validity and power of the Scientific Method!**

## The Gas Laws

- Boyle’s Law: (at constant temperature)
  - Volume is inversely proportional to Pressure.
  - The product of Pressure and Volume is a constant.
  - $P_1V_1 = P_2V_2$
- Charles’s Law: (at constant pressure)
  - Volume is proportional to Temperature
  - $V_1/T_1 = V_2/T_2$
- Combination leads to the Ideal Gas Law
  - $PV = nRT$

## Boyle’s Law 1662

- The product of the volume and pressure of a given quantity of gas at a constant temperature is a constant.  $P_1V_1 = P_2V_2$
- This is the defining property of gas as a state of matter!
- Two methods of demonstrating Boyle’s Law will be presented. Each method can be used to determine the magnitude of atmospheric pressure – an important and useful value.

## Boyle's Law – Method 1

- Method: The **pressure** on a volume of gas (~500 ml) trapped within a soda bottle will be increased/ decreased by a **known amount** using the weight of a liquid column. The **volume** changes caused by the pressure changes caused by the pressure changes will be **measured**.
- Equipment:
  - 20 oz (591 ml) plastic soda bottle w/ cap
  - ~ 4 ft (~120 cm) clear, flexible plastic tubing (~1/8" ID)
  - 60 ml plastic syringe, graduated
  - ~150 ml colored water
  - Meter stick (or yardstick)
  - Insert ~9" of the plastic tubing through a 1/4" hole in the bottle cap and seal (E6000, Goop, etc.)

## Boyle's Law Method 1 Equipment



## Boyle's Law - Initial

- Fill the bottle to a predetermined 500 ml mark. Fill the plastic tubing and syringe to ~ 30 ml. Tighten the cap on the bottle.
- With the liquid level in the syringe the same as that in the bottle, record the syringe volume as  $V_{init}$  and the Pressure as  $P_1$

$$V_1 = 500 \text{ ml}$$

$$V_{init} = 24 \text{ ml}$$

$$P_1 = \text{Atmos.}$$



## Boyle's Law Increased Pressure

- Raise the syringe ~ .5m above the bottle. Use students or fixturing to hold the syringe, bottle and meter stick so that measurements of the distance between liquid levels and the syringe volume can be made.
- Record the syringe volume as  $V_+$  and the distance between the liquid levels of the bottle and the syringe as  $D_+$ .
- Note that the liquid level in the syringe falls as it is raised above the bottle. The **weight of the liquid in the tube is compressing the gas** in the soda bottle.

## Boyle's Law Increased Pressure



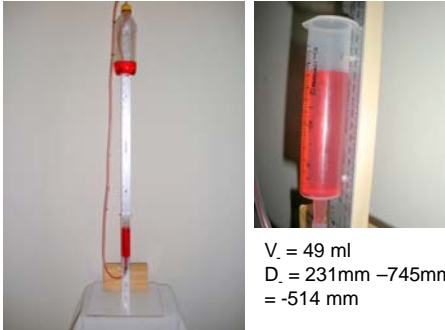
$$V_+ = 2 \text{ ml}$$

$$D_+ = 540 \text{ mm} - 52 \text{ mm} = 488 \text{ mm}$$

## Boyle's Law Decreased Pressure

- Lower the syringe approximately .5 m below the bottle. Again, students or fixturing may be used to hold everything while measurements are made.
- Record the syringe volume as  $V_-$  and the distance between the liquid levels as  $D_-$ .
- Note that the volume of liquid in the syringe has increased as the **gas in the bottle expanded due to the weight of the liquid in the tubing** "pulling" on it.

## Boyle's Law Decreased Pressure



## Boyle's Law - Data Reduction

- Initial condition
    - Pressure =  $P_1 = \text{Atmospheric}$  (as measured in liquid height)
    - Volume =  $V_1 = 500\text{ml}$  (as previously marked)
  - Increased Pressure condition
    - Pressure =  $P_2 = \text{Atmospheric}(P_1) + D_1 (+488 \text{ mm})$
    - Volume =  $V_2 = 500 \text{ ml} - (V_{\text{init}} - V_1) = 500 - (24-2) = 478 \text{ ml}$
  - Decreased Pressure condition
    - Pressure =  $P_3 = \text{Atmospheric}(P_1) + D_3 (-514 \text{ mm})$
    - Volume =  $V_3 = 500 \text{ ml} - (V_{\text{init}} - V_1) = 500 - (24-49) = 525 \text{ m}$
- $P_1 = \text{Atmosphere}, \quad V_1 = 500\text{ml}$   
 •  $P_2 = P_1 + 488\text{mm}, \quad V_2 = V_1 - 22 \text{ ml} = 478 \text{ ml}$   
 •  $P_3 = P_1 - 514\text{mm}, \quad V_3 = V_1 + 25 \text{ ml} = 525 \text{ ml}$

## Boyle's Law - Algebra

- The value of atmospheric pressure can be calculated from the previous measurements.
- As the pressure increases are being measured in mm of water column height, the apparent value of atmospheric pressure will be in units of mm of water.
- Derivation – Increased Pressure
  - $P_1 V_1 = P_2 V_2$
  - $P_1 * 500 = (P_1 + 488) * (478)$
  - $P_1 * 500 = P_1 * 478 + 488 * 478$
  - $P_1 * 500 - P_1 * 478 = 488 * 478$
  - $P_1 * 22 = 488 * 478$
  - $P_1 = (488 * 478) / 22 = 233264 / 22 = 10,603 \text{ mm}$
- Reduced Pressure
  - Same derivation to get to  $P_1 = (514 * 525) / 25 = 10,794 \text{ mm}$
- Actual value of atmospheric pressure expressed as the height of a water column is 10,300 mm (10.3 m, ~34 ft)

## Atmospheric Pressure – another way

- Magdeburg Measurement Method - 1672



Fig. 3. Hermann Nill (from von Guericke's book of 1672). Atmospheric pressure measured copper hemispheres apart by applying weights. (Courtesy of the "Otto-von-Guericke" University, Magdeburg, Germany.)

## Boyles Law – Method 2

- Method – A known volume of gas (20 ml) contained within a 60 ml syringe will expand when the syringe plunger is withdrawn by hanging known weights from it.
- Equipment
  - 60 ml syringe with the plunger modified to accept a hook
  - A plug for the tip of the syringe
  - A support for the syringe (board with notch)
  - A sturdy shopping bag
  - ~12 water bottles .5 L (~502 g each)

## Boyle's Law Method 2 Equipment,



## Boyles Law #2

- Support the syringe vertically with the shopping bag hanging from the syringe. Insure the bag bottom is at least 8" above the floor.
- Adjust the syringe plunger to 20 ml and plug the syringe tip.



## Boyles Law #2 Add weights

- Record the initial volume (20ml)
- Add water bottles to the bag one-by-one, allowing the plunger to reach equilibrium after each addition.
- Record the number of bottles and the syringe volume after each addition.

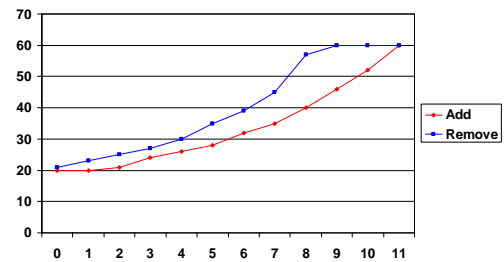
Number	Volume
0	20 ml
1	20ml
2	21ml
3	24ml
4	26m
5	28ml
6	32ml
7	35ml
8	40ml
9	46ml
10	52ml
11	60ml

## Boyles Law #2, Remove weights

- Now carefully remove the water bottles one-by-one, allowing the plunger to reach its equilibrium position after each removal.
- Again record the number of bottles still in the bag and the plunger position after each removal.

Number	Plunger
11	60 ml
10	60ml
9	60ml
8	57ml
7	45m
6	39ml
5	35ml
4	30ml
3	27ml
2	25ml
1	23ml
0	21ml

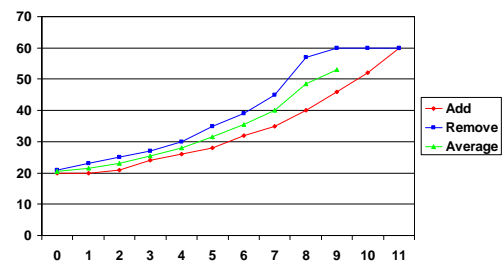
## Boyle's Law #2 Data Plot



## Boyle's law #2 Graphical Analysis

- Note that there are two different traces, one when the weights are being added, one when they are being removed.
- The plunger volume is dependent on whether the plunger is moving up or down – hysteresis due to the plunger friction.
- One method to compensate for this type of experimental bias is to average the values.

## Boyle's Law #2 Data Plot with Average



## Boyle's Law #2 Atmos. pressure est.

- Observe that the volume in the syringe doubles from 20 ml to 40 ml (average) when 7 bottles are hanging from the plunger. Since the volume is twice what it was, the pressure must be  $\frac{1}{2}$  of what it was (atmosphere)
- The syringe diameter is measured at 2.67 cm.
- Knowing the area of the plunger ( $\pi * (2.67/2)^2 = 5.59$ ) and the weight, ( $7 * 502g$ ) allows the estimation of atmospheric pressure  $2 * (7 * 502) / 5.59 = 1257$  grams-force/cm<sup>2</sup> (a bad SI unit).
- The actual value of atmospheric in these units is **1032** grams-force/cm<sup>2</sup>.

## Charles's Law - Temperature effect

- The volume of a quantity of gas (at a constant pressure) is proportional to it's temperature  $V_1 / T_1 = V_2 / T_2$  where T is on a absolute temperature scale.
  - This implies that at T = 0 the volume of the gas becomes 0, either it vanishes or becomes infinitely dense!
- This behavior is a property of the state of matter - gas, not the specific material such as it is for solids and liquids.
- A method for monitoring and measuring the volume change induced by cooling and/or heating a fixed quantity of gas will be demonstrated.

## Charles's Law Demonstration

- Method: Gas within a sealed bottle is heated or cooled and the resultant volume change is measured. The volume measurements are made such that the pressure of the gas remains constant.
- Equipment:
  - 20 oz (591 ml) plastic soda bottle w/ cap
  - ~ 4 ft (~120 cm) clear, flexible plastic tubing (~1/8" ID)
  - 60 ml plastic syringe, graduated
  - ~150 ml colored water
  - Thermometer if an estimation of Absolute Zero is desired.
  - Container for ice or hot water – large enough for bottle.
  - Insert ~9" of the plastic tubing through a 1/4" hole in the bottle cap and seal (E6000, Goop, etc.)

## Charles's Law Equipment



## Charles's Law Initial

- Fill the bottle to a predetermined 500 ml mark. Fill the plastic tubing and syringe to ~ 50 ml if the bottle is to be cooled or ~10 ml if warm water is used. Tighten the cap on the bottle.
- Submerge the bottle in the container filled with room temperature water.

- With the **liquid level in the syringe the same as that in the bottle**, record the temperature as  $T_{init}$  and the syringe volume as  $V_1$

$$V_{init} = 500\text{ml}$$

$$T_{init} = 21.5^\circ\text{C}$$

$$V_1 = 53\text{ml}$$



## Charles's Law - Ice

- Dump the room temperature water from the container and replace with a water/ice mixture.
- Again submerge the bottle and allow it to come to thermal equilibrium (~5 min.)

- With the liquid level in the syringe the same as that in the bottle, record the temperature as  $T_{Cold}$  and the syringe volume as  $V_2$
- $$T_{Cold} = 3.2^\circ\text{C}$$
- $$V_2 = 18\text{ml}$$
- $$V_{Cold} = 500 - (53 - 18) = 465\text{ml}$$



## Charles's Law – Warm

- The previous demonstration can be performed using warm water (~40°C). Be sure to start with only ~10 ml in the syringe to allow for the expected expansion.

$$T_{\text{Init}} = 21.0^{\circ}\text{C}$$

$$V_1 = 7 \text{ ml}$$

$$T_{\text{Hot}} = 37.9^{\circ}\text{C}$$

$$V_2 = 52 \text{ ml}$$

$$V_{\text{Hot}} = 500 + (52 - 7) = 535 \text{ ml}$$



## Charles's Law – Data Reduction

- The zero point of the Celsius temperature scale is arbitrarily set at the freezing point of water. Temperature measurements can be put on an absolute scale by adding the temperature difference between Celsius zero and the value of absolute zero.
  - Thus  $T_{\text{Abs}} = T_{\text{Celsius}} + T_0$  where  $T_0$  is the value of absolute zero (in Centigrade).
  - Room temp.:  $T_R = (21.5 + T_0)$ ,  $V_R = 500 \text{ ml}$
  - Cold:  $T_C = (3.2 + T_0)$ ,  $V_C = 500 - (53 - 18) = 465 \text{ ml}$
  - Hot:  $T_H = (38 + T_0)$ ,  $V_H = 500 + (42 - 7) = 535 \text{ ml}$

## Charles's Law – Derivation of $T_0$

- The value of absolute zero can be estimated from the previous "Cold" measurements.
  - $V_{\text{Init}} / (T_0 + T_{\text{Init}}) = V_{\text{Cold}} / (T_0 + T_{\text{Cold}})$
  - $500 / (T_0 + 21.5) = 465 / (T_0 + 3.2)$
  - $T_0 = ((465 * 21.5) - (500 * 3.2)) / (500 - 465) = 240^{\circ}\text{C}$
- The "Hot" measurements can also be used to estimate the value of absolute zero
  - $T_0 = ((535 * 21) - (500 * 38)) / (500 - 535) = 222^{\circ}\text{C}$
- The actual difference between 0°C and absolute zero is **273.15°C**

## Charles's Law – Gas Thermometer

- The objective is to demonstrate a practical application of Charles's Law by constructing a highly sensitive (resolution ~ .005°C !) gas thermometer.
- Method:
  - Gas contained within a closed bottle "pushes" a liquid into a tube having a relatively small diameter. Small changes in the gas volume due to temperature changes result in large changes in the liquid level.
- Equipment:
  - The same equipment as used in the Boyle's Law Method 1 demonstration is used.
  - The difference being that the plastic tube is attached to a vertical meter stick so that the height of the liquid column can be measured.

## Gas Thermometer - Assembly



## Gas Thermometer - Demonstration

- With the plastic tubing held vertically above the sealed soda bottle, inject ~30 ml of air into the bottle through the tube – use the 60 ml syringe.
- When the syringe is removed, the "extra" gas in the bottle will push the liquid up into the tube so it can be measured.
- You can now either heat the bottle a little by cupping your hands around it or cool it with a little cold water or canned air.

## Gas Thermometer - Calculations

- Assuming the bottle contains ~500 ml of gas, the expected gas expansion is  $500/300 = 1.7$  ml per degree Centigrade.
- The plastic tubing has an ID of  $\sim .16$  cm ( $\sim 1/16$ " ) producing a cross section of  $.08$  cm<sup>2</sup>. Dividing  $1.7$  ml by  $.08$  cm<sup>2</sup> produces an expected length of  $21.25$  cm.
- A 1mm change in the liquid level would then be equivalent to  $1/212.5 * 1^{\circ}\text{C} = .0047$   $^{\circ}\text{C}$ !
- **NOTE – The actual change in the liquid level will be significantly less than this calculation indicates! WHY?**

## Gas Thermometer - Real World

- Why the gas thermometer does not behave strictly as Charles's Law would predict.
  - The rising liquid level in the tube produces an increased pressure within the bottle. Boyle's Law indicates that the increased pressure would reduce the volume of the gas.
  - The actual liquid level rise results from a competition between the expansion produced by Boyle's Law and the contraction produced by Charles's Law.
  - Each law is only accurate when one variable (either T or P) is held constant!
  - The "expected" liquid level increase can still be calculated using both Boyle's Law and Charles's Law but the algebra gets pretty messy.

## Presentation Summary

- An appreciation of the Gas Laws is critical to understanding many of the technologies surrounding our lives.
- The Gas Laws can be explored and understood using simple, cheap, and readily available equipment.
- Exploration of the Gas Laws can provide an ideal setting for the introduction of many of the tools and techniques of science:
  - Observation and measurement
  - Data analysis and plotting
  - Algebraic manipulation and parameter solving
  - Real world experimental limitations, errors and artifacts.

## Gases, Vacuum and more

- Please stop by the NCCA VS EdCom table in the exhibition area (Table G) to learn more about our programs and projects.
  - Science in Action – K through 12 classroom presentations of vacuum effects and technology(SF Bay area only at present)
  - Science Educators Workshop – Annual sponsorship of selected teachers to attend the National AVS Symposium and receive a complete set of vacuum equipment and training.
  - Science Fair Judges and Awards – we provide judges and awards at selected SF Bay area science fairs.
- This presentation and many more resources are available at our web site:
- **<http://www2.avs.org/chapters/nccavs/edcom.html>**